PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A
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A Compensated Interference Dilatometer

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VIII. *A Compensated Interference Dilatometer.*

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The instrument now described is a form of Fizeau interference dilatometer, in which the sensitiveness of the interference method is so largely increased as to render it unnecessary to employ a plate or block of the solid, whose expansion is to be measured, of greater thickness than 5 millimetres. The author has been led to adopt it for the accurate determination of the thermal expansion of the crystals of artificial chemical preparations, which it is frequently impossible to obtain sufficiently large, and at the same time homogeneous, to furnish plates a centimetre thick, as demanded by the forms of apparatus hitherto described.

The exquisite method devised by Fizeau ('Compt. Rend.,' vol. 58, p. 923, and vol. 62, p. 1133; 'Ann. Chim. Phys.,' [4], vol. 2, p. 143, and [4], vol. 8, p. 335) depends essentially upon the determination of the difference of expansion, which accompanies rise of temperature, between the screws of a small metallic tripod and the object under investigation which is supported by it. In the form of apparatus now described the expansion of these screws is compensated and eliminated, thus rendering the total expansion of the object available for measurement. Hence, it is possible to obtain a result with a small crystal of as satisfactorily accurate a character as was formerly only to be obtained with a much larger crystal. Besides the introduction of this compensating principle, the new instrument combines, in the author's opinion, the best features of the several forms of Fizeau dilatometer previously described; at the same time it is essentially different from any one of these previous forms, and includes many details of a novel character. The author is largely indebted to the Memoirs of Benoit ('Trav. et Mémoires du Bureau Int. des Poids et Mesures,' vol. 1, 1881, p. 1, and vol. 6, 1888, p. 106), concerning the Fizeau apparatus belonging to the Bureau International des Poids et Mesures, Paris, and the classical work which he has accomplished by the use of it; and to the later one of Pulfrich ('Zeitschrift für Instrumentenkunde,' 1893, p. 365) concerning an improved form of apparatus embodying the important modifications introduced into the method by Abbe in the year 1884 and described by Weidmann in 1889 ('Wiedemann's Annalen,' vol. 38, p. 453).

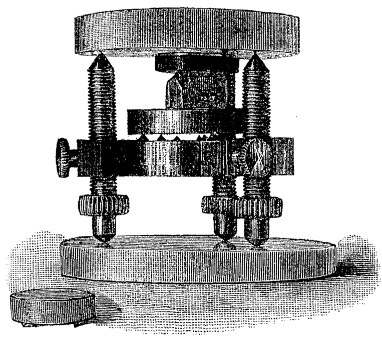
Description of the Apparatus.

The Interference Tripod.

The first essential of an interference dilatometer is the tripod support for the object and for the glass lens or plate to be suspended, without touching, above the latter. The upper surface of the object and the lower surface of the glass are the two surfaces, by reflection of pure monochromatic light from which interference curves or bands are to be produced; and it is the movement of these bands upon alteration of the temperature which affords the measurement of the change in thickness of the air space which separates the two surfaces. For the passage of two successive curves or bands past any given spot determines precisely that a change of thickness equal to half a wave-length, of the particular light employed, has occurred during the interval.

The tripod employed by Benoit was constructed of platinum-iridium; that of Abbe and Pulfrich was of steel. The tripod of the apparatus now described is constructed of platinum-iridium, which, in addition to its obvious advantage on the score of unalterability, and its exceptionally small coefficient of expansion, is specially suitable for a reason which will be apparent when the method of compensation is described. It is represented in fig. 1.

Fig. 1.



It is entirely constructed out of a single homogeneous casting of the alloy of platinum with 10 per cent. of iridium, which was supplied by Messrs. Johnson and Matthey for the purpose, in order that the thermal expansion should be uniform throughout its whole extent. It consists of a solid horizontal table 7 millims. thick, circular in shape, but with three short equi-distant projecting arms, through which are threaded the three screws, of very fine pitch, each 35 millims. long, upon the bluntly-pointed upper ends of which the glass cover-wedge is to be supported. The distance between the centres of any two adjacent screws is 30 millims. The screws are provided, near their lower rounded ends, with milled heads for convenience of rotation; and each of the arms is slit from one side, the slit passing through the centre of the screw hole and slightly beyond the latter, and provided with a small clamping screw, which draws the two sides of the slit together, binding the main vertical screw and thus enabling it to be firmly fixed in any position. Both sides of the table are polished, and on one side three concentric series of raised points are provided, three equi-distant ones in each series, and each series increasing in height as the centre is receded from. These serve as rests for objects of different sizes.

The Compensation.

It is an interesting fact that pure aluminium expands 2·6 times as much as platinum-iridium for the same increment of temperature, and it is this fact which the author has utilised for the purpose of effecting compensation for the expansion of the platinum-iridium screws. A circular disc of aluminium, whose thickness is about ten twenty-sixths of the length of the parts of the platinum-iridium screws projecting through and above the table, is laid upon the latter. Provided adequate care has been taken to adjust the screws to the length calculated for complete compensation, from a knowledge of the two coefficients of expansion, it will be evident that the expansion of the apparatus is entirely eliminated. The length of screw provided affords, after compensation, an available space of some millims. above the aluminium block in which to place the crystal or other substance to be investigated, and as this space remains unaltered by change of temperature, any alteration of the thickness of the air layer between the upper surface of the object and the lower surface of the glass disc laid upon the screws must be entirely due to the expansion of the crystal. Moreover, the relative positions of the aluminium block and the crystal may be interchanged, it being immaterial which is uppermost. For the case in which the crystal rests on the block, a series of circular aluminium blocks of 25 millims. diameter and of respectively 12, 10, 8, 6 and 4 millims. thickness are provided, suitable for use with all the various sizes of crystals which are likely to be met with. The thinnest one is shown in position in fig. 1, resting upon the raised points. They were constructed out of a homogeneous casting of the purest aluminium obtainable. It is only necessary to select the block which is most suitable for use in connection with the particular size of crystal to be employed, and to set the screws to the proper corresponding length, which has been calculated, verified experimentally, and recorded in a table once for all.

The polish which many crystalline solids take upon their ground plane surfaces is rarely of a character equal to that of glass, and it is particularly desirable that the two reflecting surfaces relevant to the interference should reflect as equally as possible, in order that the interspaces between the bands shall be as dark as possible. Moreover, it is likewise desirable that the lower reflecting surface should be fairly extended, in order to afford an ample field of interference bands; and that a crystal

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should not have to be rejected as unsuitable because either one or both of the parallel horizontal faces, perpendicular to which the expansion is to be measured, is, relatively to the other dimensions of the crystal, small. For these reasons the author usually lays upon the crystal a circular plate of glass, sufficiently thick to be adequately rigid, the lower surface of which is ground dull and the upper polished and worked perfectly plane. The glass employed is black astronomical glass, in order that no disturbance may arise from an illuminated lower surface, as would be more or less the case with colourless glass. The expansion of this glass, which has been accurately determined, is very nearly the same as that of platinum-iridium, so that the very slightly corrected measure of the thickness of the plate has simply to be added to the length of the screws calculated for compensation by the aluminium, and the screws set to this total length. Three series of such discs, of respectively 25, 15, and 10 millims. diameter, have been provided for use with crystals of greater or less lateral extension; for it is unwise to employ the 25-millim. disc with crystals of small diameter, a columnar type placed on end for instance, and the smaller ones still provide an adequate field of bands. One of medium size is shown resting on a crystal in fig. 1. Each series consists of three plates, varying from 2 to 4 millims. in thickness, so that there is ample choice afforded. Such a choice is preferably made as will leave, when the crystal and compensator are in position, an air space, between the upper surface of the glass plate and the lower surface of the large disc resting on the screws, of less than half a millimetre, in order that brilliant interference bands may be obtained. The size mostly employed by the author with crystals of artificial salts is 10 millims. diameter and 2 millims. in thickness.

[The second method of using the compensator, namely, above the crystal, is the one which the author prefers to employ whenever possible. Three series of aluminium discs are provided for use with this method, of respectively 15, 10, and 6 millims. diameter. Each disc is polished as truly plane as possible on one surface, intended to be the upper surface when in position on the crystal, and carries on the other side three equi-distant points, the ends of miniature screws of the same aluminium firmly screwed into the disc. Each series consists of two discs, the thicknesses of which, including the points, are respectively 4 and 5 millims. One of these compensators is shown resting on the ground in fig. 1. By this method the lower surface of the crystal rests directly on the three points of the platinum-iridium table, the particular three varying with the size of the crystal but being usually the innermost three, and the aluminium compensator rests by its three points upon the upper surface of the crystal. Hence any movement due to very slight convexity of the surfaces, or possible rolling upon a speck of dust included between the surfaces, is entirely avoided by this arrangement of three-point contact only. Moreover, the calculations are simplified and sources of error reduced by avoiding the use of a glass crystal-covering disc. In this connection it is particularly fortunate that aluminium does not take a very high polish, but one which happens to reflect light about equally with the lower surface of the glass cover-wedge laid on the screws, and thus affords most excellent bands. The smallest compensators, of 6 millims. diameter, still exhibit a field of about eight bands of normal width, ample for the purpose of the observations; the points of these compensators are only 5 millims. apart, so that the method is applicable down to crystals of only slightly greater extent of surface than is adequate for these points to rest upon.—May, 1898.]

The Cover-Wedge.

The thick glass disc of 40 millims. diameter, which is placed over the screws of the tripod, and whose lower plane surface is to form the upper of the two surfaces relevant to the production of interference, is not lenticular, as employed by Fizeau and by Benoit, but is possessed also of an upper plane surface, as used by Abbe and by Pulfrich, in order that parallel rectilinear bands and not curved ones may be produced. The surfaces of this plate are not precisely parallel, but are inclined at an angle of thirty-five minutes, forming in reality a wedge of extremely small angle. By this device the reflection of the illuminating light from the upper surface, which would otherwise illuminate the interspaces between the bands, is deflected out of the field of vision.

General Arrangement of the Dilatometer.

The interference apparatus which has now been described is supported during the observations in a manner similar in principle to that employed by Abbe, but differing in the constructive details; and the heating apparatus is an air bath of special construction instead of an oil bath. Moreover, the interference tripod rests on non-conducting material in direct contact with the heated air, whose temperature is measured by the bath thermometers, and its actual temperature is determined by a third thermometer, so bent that the bulb is in direct contact with it. Further, this portion of the dilatometer, which may be termed the expansion apparatus, is, unlike the Abbe arrangement, separated from the illuminating and observing apparatus, which latter is removed to a considerable distance in order to be well out of the range of the heated atmosphere in the neighbourhood of the air bath.

A general view of the dilatometer and its accessories is given in fig. 2. The two parts, the expansion apparatus on the left, and the illuminating and observing apparatus on the right, are mounted at the two ends of a rigid slate table six feet long.

The Expansion Apparatus.

The details of the expansion apparatus will be more readily understood with the aid of the section, fig. 3. The tripod, a, stands upon a thick circular glass plate, b, of 50 millims. diameter, which forms the floor of the interference chamber. This

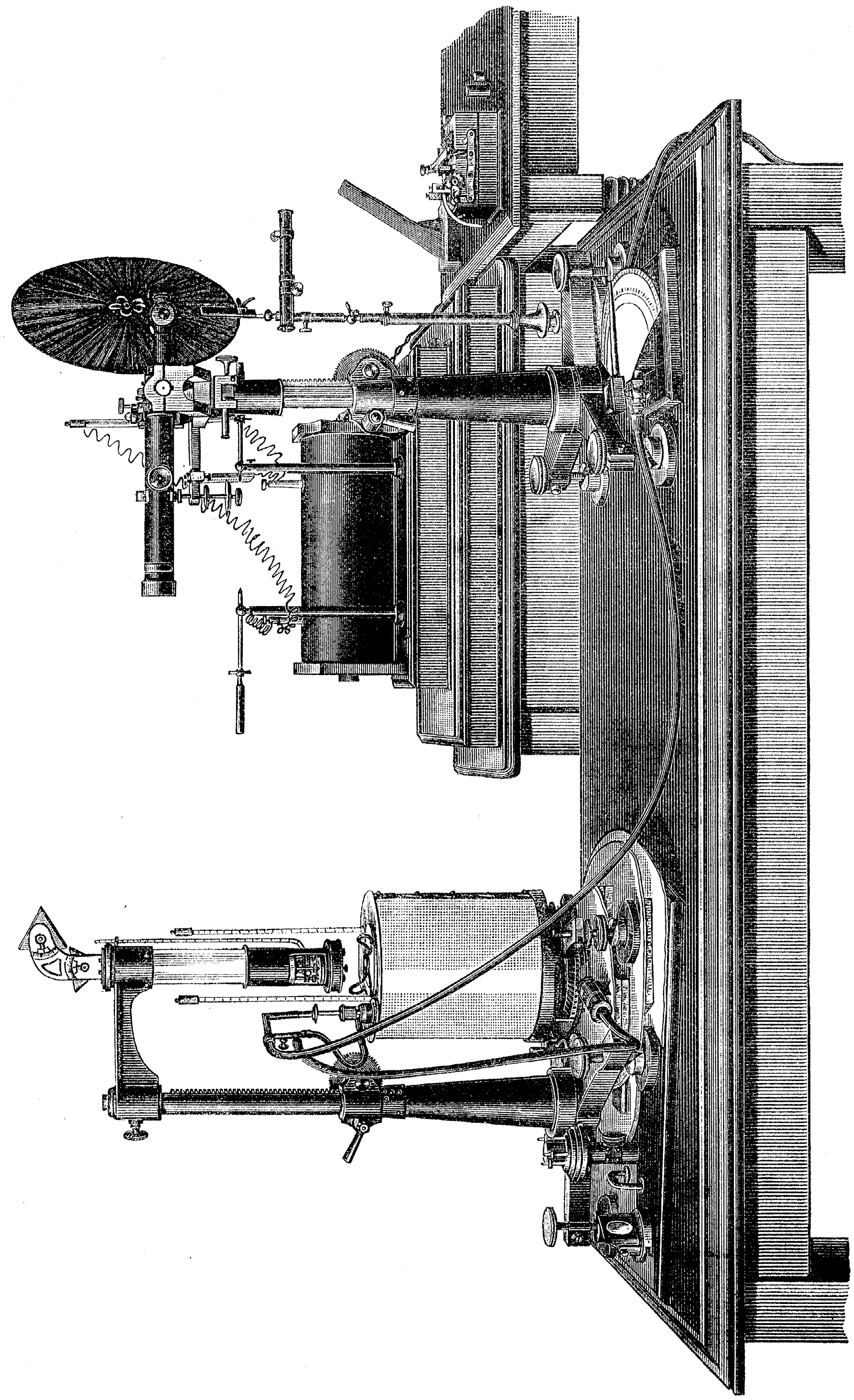
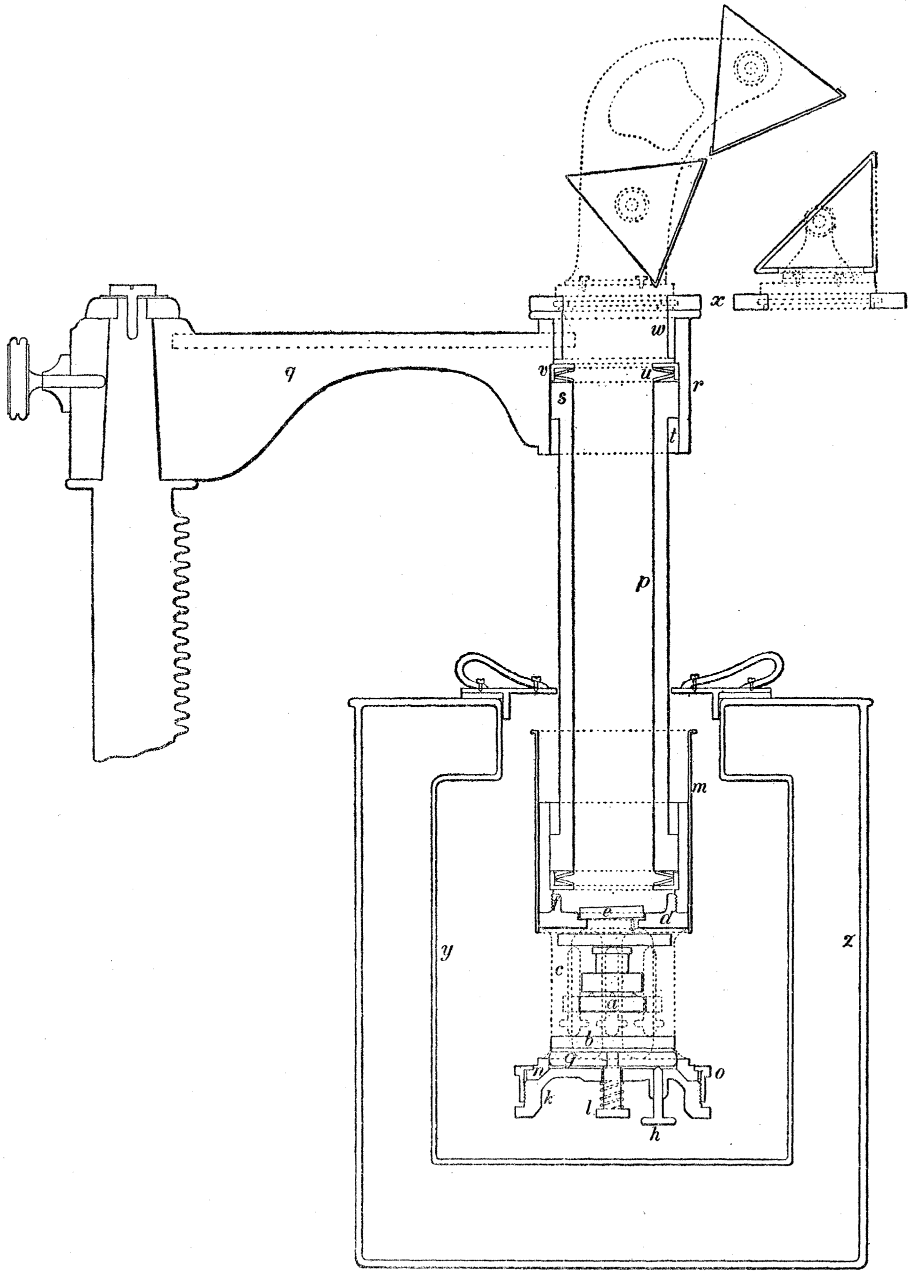


Fig. 2.

chamber consists of a wide gun-metal tube, *c*, very little longer than its diameter, and the cylindrical wall of which is so considerably cut away in three equi-distant parts as to form three relatively large windows separated by three equi-distant

Fig. 3.



pillars adequate to maintain rigidity. It is closed at the top by a stout diaphragm, *d*, whose aperture is filled by a circular glass plate, *e*, 27 millims. in diameter, which serves the double object of acting as a non-conducting roof to the interference chamber, to prevent upward radiation and consequent loss of heat into the supporting tube, and of correcting the slight prismatic deviation of the light rays introduced by making the cover-wedge slightly wedge-shaped. The correction is achieved by making the disc, *e*, equally wedge-shaped. In fact, the two discs are cut from the same piece of plate glass, which has been ground down so that the two perfectly plane surfaces are 35 minutes out of parallelism. The direction of the wedge is marked by a straight line engraved parallel to the long edge of the slab before cutting the discs from it. The latter are so cut that this engraved line comes near the margin of each, forming a chord of the circle. It is only necessary to place the two discs so that the engraved chords are parallel and on the same side of the centre, but so that the marked surface of the larger disc of the interference apparatus lies uppermost while that of the smaller diaphragm disc lies underneath, when the refractive effect is fully counteracted. In order that no reflections shall be visible due to the two surfaces of the counteracting diaphragm wedge, the latter is slightly tilted by means of two very small screws driven through the rabbet of the aperture from the under side in such positions that the line joining them is parallel to the engraved chord of the wedge; when the disc is laid in its rabbeted aperture it rests upon the two screw points and a third equi-distant point on the surface of the rabbet, the lines joining the three points of contact thus forming an equilateral triangle. The slight tilt given to the disc when the screws are properly adjusted is sufficient to throw the reflected rays so far out of the optical axis of the system as to be no longer visible through the micrometer eyepiece.

The chamber terminates above the diaphragm roof in a screw thread, *f*, which engages with one on the lower end of the supporting apparatus. Below, the chamber ends in an adjustable table, *g*, upon which rests a disc of asbestos millboard, which, in turn, supports the glass floor disc. The adjustment is effected by three milled-headed screws, *h*, worked from underneath, which raise or lower the table above the rigid gun-metal base, *k*, of the chamber through which the screws pass. This rigid base is also perforated by a central hole, through which passes a short cylindrical rod fixed to the under side of the adjustable table; the rod terminates in a boss, *l*, between which and the under side of the fixed base a spring is confined, which causes the adjustable table to be always pulled firmly down upon the ends of the three screws. The chamber can be closed when desirable, in a light-tight manner, by a concentric outer tube, *m*, capable of sliding over it from above, with sufficient friction to maintain it up when the chamber is required to be open during adjustment of the interference apparatus. The author removes it altogether when the interference chamber is immersed in the air bath for the purpose of observations at the higher temperatures. The rigid base of the chamber and its adjustable table may be readily detached from the chamber if required, but the windows are sufficiently large to admit the tripod without this necessity arising merely for that purpose. To provide for its occasional desirability, however, the lower end of the chamber wall terminates in an annulus and flange, *n*, which rest upon a corresponding rabbet cut in the rigid base ; rotation is prevented by a slot and pin. The rigid base and the flange are bound together by means of a large milled nut, *o*, which has above an inner flange projecting over the chamber flange just referred to, and whose thread engages with one turned in the periphery of the base.

The interference chamber is supported at the lower end of a non-conducting tube of Berlin porcelain, *p*, supported from an arm, *q*, carried by a stout gun-metal columnar pedestal with three-legged base, provided with levelling screws resting on toe-plates. The column is provided with a strongly-made vertical adjustment by rack and pinion, worked by a milled ebonite disc, and provided with an arresting clamping screw, manipulated by a lever to securely fix it at the required height. Sufficient rack is given to enable the interference chamber to be raised above the height of the air bath, and conversely to be readily lowered into the latter. Particular care has been taken to render the working of the rack smooth and rigid, in order that no movement of the adjusted interference apparatus shall occur during the operation. The arm is rotatable about the upper part of the column, and can be fixed when desirable by a clamping screw. The outward end of the arm carries, as part of the same casting, a short thick-walled tube, *r*, of the same diameter as the interference chamber ; at the lower end of this tube the porcelain tube is supported. In order to be prepared for the possible occasional fracture of a porcelain tube during heating, half-a-dozen such tubes were specially prepared, from the author's pattern, at the Berlin porcelain works. They are glazed outside and biscuit within, and have each a flange at either end ; by subsequent trimming in the lathe the flanges have all been reduced exactly to the same size, so that a fractured tube can be readily replaced by another. This is also facilitated by the method of suspension, which is likewise one that renders fracture very improbable, as it admits of freedom of expansion. The flange, *s*, passes easily up the metal supporting tube carried by the arm, and after its insertion the two halves of a collar, *t*, are passed up the metal tube under the flange until they are flush with the lower end of the tube, when they are secured by screws passing through the tube and collar from the outside. In order that the porcelain tube, thus easily suspended, may be sufficiently rigid for the purpose in view, slight pressure is brought to bear on the flange from above by a pair of circular bent springs, *u*, confined between the flange and another internal collar, *v*, in the upper part of the metal tube. The lower flange of the porcelain tube is similarly attached to a short metal tube, which carries inside, at its lower end, an adequately long screw thread, which engages with the one at the upper end of the interference chamber, by which means the latter is attached.

The incident light rays are directed into the expansion apparatus from the illuminating apparatus, and the reflected rays back into the latter (which, being arranged for autocollimation, serves also as observing apparatus) by means of one of two interchangeable pieces of deflecting apparatus. The first is a single total reflecting prism ; this is employed during the preliminary adjustment of the interference apparatus, with the aid of ordinary white light, and also when it is only desired to produce the interference bands by monochromatic flames. The second is a train of two flint glass refracting prisms, which is employed for the production and observation of the bands produced, with the aid of a Geissler tube containing rarefied hydrogen and mercury, by the monochromatic light corresponding to the red and greenish-blue hydrogen lines, C and F of the spectrum, and to the green mercury line found so suitable by PULFRICH. The interchangeability of the two is very simply attained. In the top of the tubular termination of the arm, above the internal collar, there fits a very much shorter tube, w, terminating above in a stout flange; it is capable of rotating, for the purpose of azimuth adjustment, in the main tube, without the possibility of vertical motion, by means of an annular groove, into which project a couple of screws driven through the outer tube from opposite sides. The flange, whose central aperture is of slightly greater diameter than that of the porcelain tube, carries at two opposite sides raised and grooved guides, in which are capable of sliding the correspondingly rabbeted basal annular supports, x, of the single or double prism arrangements.

The single reflecting prism is supported in suitable bearings by central axles rigidly attached to the metallic case, and provided with milled bosses at the ends for convenience of rotating the prism; this enables the latter to be adjusted for altitude, and as the axles fit fairly tightly in the bearings, the setting remains unaltered after adjustment.

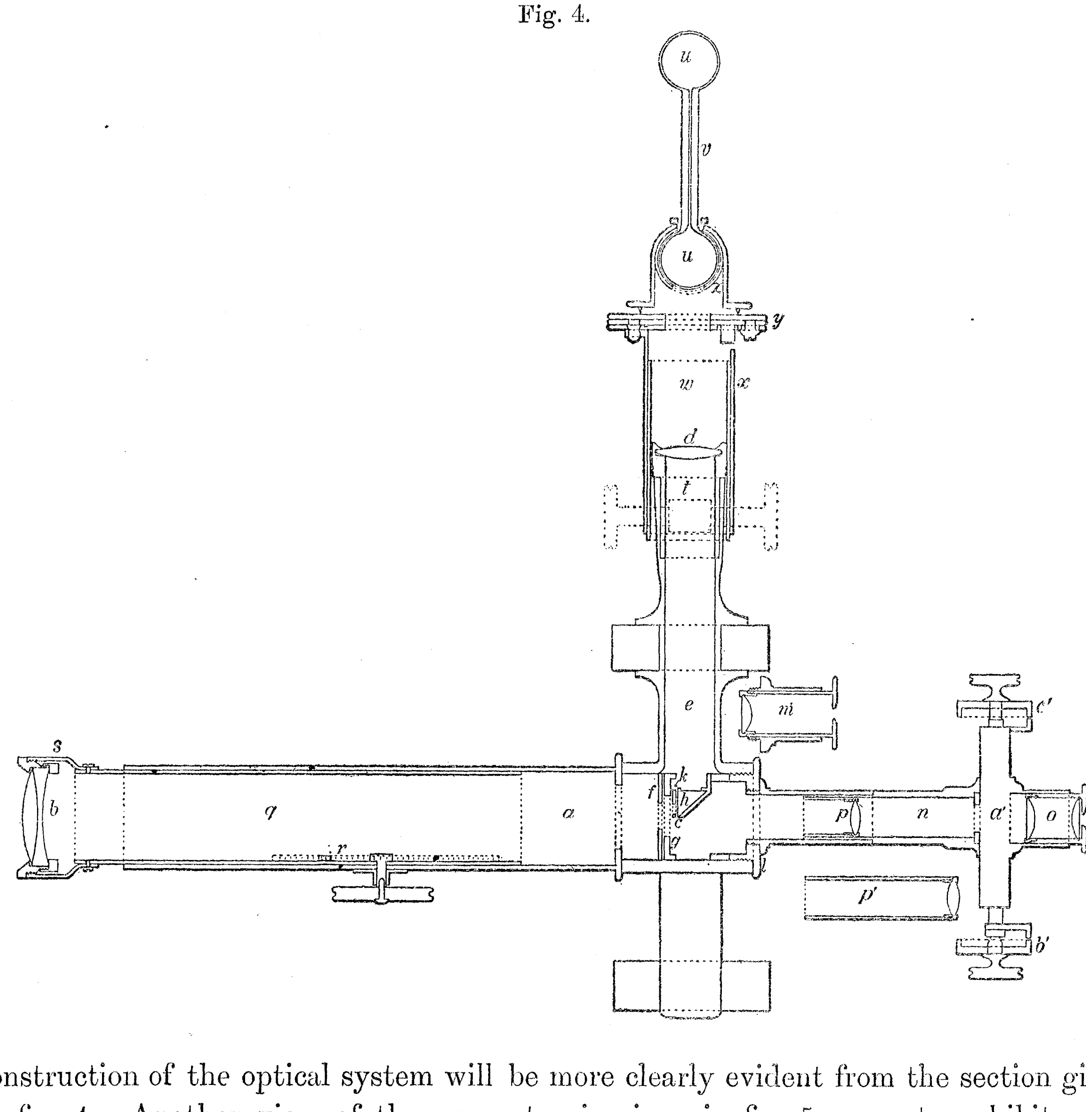
The pair of refracting prisms are mounted in a light framework, whose shape is calculated to ensure rigidity. Each prism is cased in metal, and the case carries centrally at its two sides short but thick axles, narrowing somewhat after a distance equal to the thickness of the mount, and terminating in a longer screw thread. The axles gear at their thickest part in corresponding bearings in the supporting framework. Over the two projecting axial screws on one side of the mount are passed in each case first an arm carrying at its end a silver arc as if for a vernier, and which slips over the screw and narrower portion of the axle only and is arrested at the thickening, and then a milled nut engaging with the thread, by which the arm may be clamped firmly to the axle and hence to the prism. As the thick part of the axle projects a little beyond its bearing, however, the arm is not clamped to the mount. The silver arc carries a central indicating mark, and by rotation of the milled head and the prism travels over a divided silver quadrant carried by the mount, and thus the prism may be set to any angular reading which may at any subsequent time be reproduced. The axial screws at the other side of the mount receive in each case first a washer and then a milled nut similar to that on the other side; the thick part of the axle being in this case slightly shorter than the thickness of the frame, the prism is clamped firmly to the mount on screwing home the nut, and so the prism may be rigidly fixed to the indicated circle reading. The refracting angle of each of these prisms is approximately 57°, which in the case of the particular flint glass employed gives a total minimum deviation due to the two of 90° for the middle part of the spectrum, and of about 88° for the red hydrogen line C, and 92° for the greenish blue hydrogen line F.

The heating apparatus consists of a double-chambered cylindrical air bath, mounted on a stout annular support resting by means of three levelling screws upon toe-plates also carried by the movable slab. During adjusting operations, and observations at the temperature of the room, the bath is removed from its stand. It is constructed throughout of brazed copper, and is covered outside with a thick coat of asbestos cloth. The inner chamber, y, is similar in shape to the outer, z, from which it is separated all round by the air space of about 4 centims. Within this inner bath the interference chamber of the dilatometer is to be lowered, by means of the rack and pinion, for observations at higher temperatures. The opening in the top of the bath is necessarily a little larger than the widest part of the interference chamber, the flange of the rigid base, k (fig. 3). The inner bath is supported within the outer one by the tubular wall of the opening, $3\frac{1}{2}$ centims. deep, which connects the top of the outer with the top of the inner bath. The top of this tubular opening is closed, when the interference chamber has been lowered into position, with the two overlapping halves of an asbestos-lined and closely-fitting lid, which encircles the porcelain tube of the dilatometer, and is provided with suitable handles above and a deep rim beneath, which latter passes down into the tubular neck sufficiently far to obviate any appreciable diffusion of cold air from outside. Through a wide tubular opening at the top and near the side of the bath there is inserted into the outer bath, for the purpose of enabling the temperature within the bath to be regulated with constancy, a Muenke thermostat, provided with a regulator to control the size of the flame of the gas-burner when the mercury in the thermostat closes the orifice in the steel gas-supply tube. There are also two short tubes, on opposite sides of the centre, which pass air-tight through the tops of both baths into the inner one, for the passage of two thermometers. Both the thermostat and the thermometers are suspended free of the metal by non-conducting stoppers, and the thermometers are arranged so that the bulbs are on either side of and in close proximity to the interference tripod. A third thermometer, of such special construction that its bulb lies in actual contact with the tripod, passes through a hole in the front half of the lid of the bath. Details concerning the thermometers will be given at a later stage. A twenty-jet Fletcher ring gas-burner is employed as source of heat, the size of the jets being controlled by the thermostat, which has been adjusted so as to obtain a constant upper temperature, as near the particular desired limit as possible. Of very great utility in attaining constant temperatures has been found a gas-tap provided with a long pointed lever arm, travelling over a graduated quadrant, divided to read directly to degrees. By its aid the temperature can be very nicely regulated, leaving only such little further regulation to the thermostat as is rendered necessary by the very slight variation in the pressure of the gas supplied from a meter furnished with a governor.

Illuminating and Observing Apparatus.

The illuminating and observing apparatus consists of an autocollimating telescope, mounted separately upon a stout pedestal similar to that which supports the expansion apparatus, provided likewise with a very strong rack and pinion vertical adjustment, in order to enable the telescope to be adjusted to the height required by the particular deflecting arrangement mounted on the expansion apparatus. The

Fig. 4.



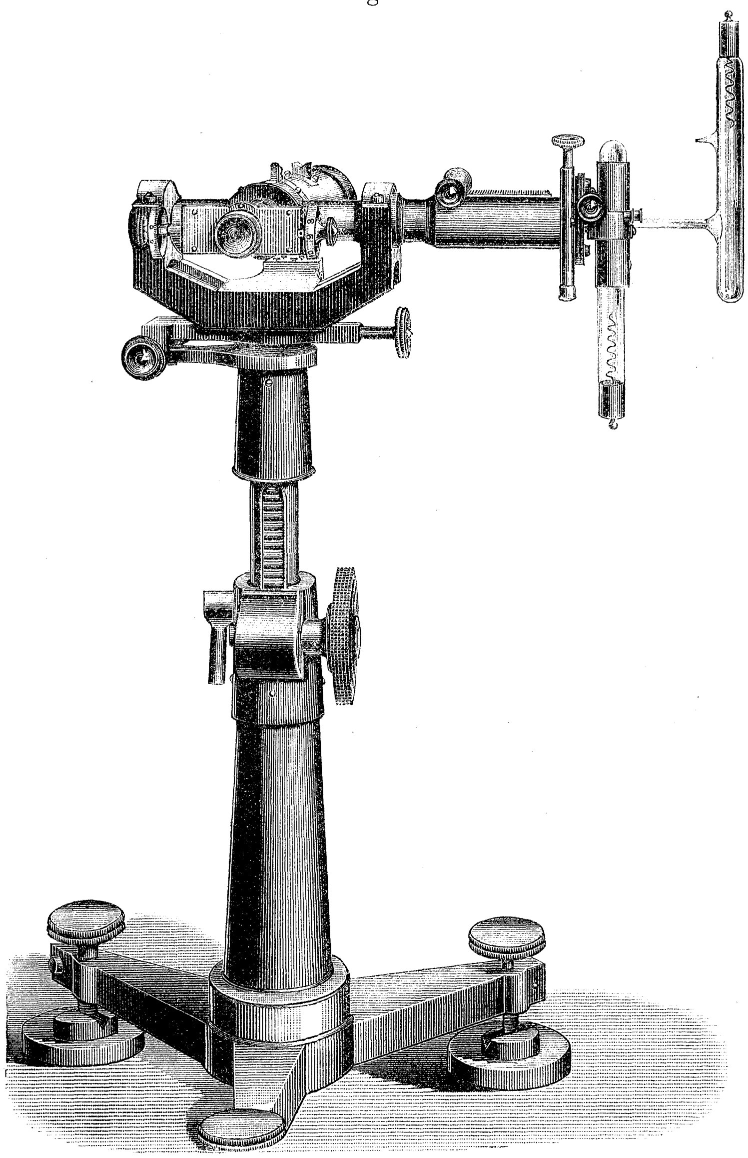
construction of the optical system will be more clearly evident from the section given in fig. 4. Another view of the apparatus is given in fig. 5, so as to exhibit more clearly the illuminating arrangement.

The main optical tube, a, carries at the end furthest from the observer an achromatic objective, b, whose focus is at c, which is likewise the focus of the rays from the illuminating lens, d, carried at the end of the side tube, e. At c is an iris diaphragm, f, immediately behind which are a fixed diaphragm, g, pierced by a relatively large aperture, and a totally-reflecting prism, *h*, covering one-half of the aperture and so arranged that the rays from the illuminating lens, *d*, are reflected through this half of the aperture towards the objective, *b*, which latter they are capable of filling with light. The surface of the prism nearest the objective is intended to be masked by one or other of a series of interchangeable stops, *k*, having larger or smaller rectangular apertures. A short distance after the prism comes a stout milled flange, *l*, to the thread within the aperture of which either of two interchangeable observing optical arrangements is capable of being attached. One, *m*, is an ordinary eyepiece, for use in adjusting the interference apparatus by observation of the images of the rectangular signal aperture, *k*, in front of the reflecting prism, reflected from the various reflecting surfaces of that apparatus. The other, *n*, is an optical combination of a micrometer two-lens eyepiece, *o*, with a third movable lens, *p*. The system is such that when the lens *p* is at that end of its path nearest the prism, the reflecting surfaces of the interference apparatus and the interference bands which they exhibit are clearly focussed to the observer looking through the eyepiece *o*, when the expansion apparatus is approached very near to the observing apparatus; and when *p* is at the end of its range nearest to the eyepiece *o*, the same result is achieved for the separation of the two parts of the apparatus by about five feet. There is no necessity for a separate adjusting apparatus, as used by PULFRICH, for when the first of the two conditions just referred to obtains, the two parts of the apparatus are so close together that it is quite easy to manipulate the screws of the interference tripod while actually observing. The relatively large windows of the interference chamber, as distinguished from PULFRICH's closed chamber, greatly facilitate this.

For convenience in moving the expansion apparatus between the two positions, the toe-plates of its pedestal do not, like those of the telescope, rest directly upon the long slate table upon which the instrument is mounted, but on a smaller rigid slab of hard mahogany, lined underneath with thick baize, and furnished with a couple of stout brass handles.

In addition to the vertical rack and pinion motion provided in the pedestal of the telescope, the latter can be adjusted for altitude by a tilting arrangement. It is mounted about trunnions and the axis of rotation is made coincident with that of the illuminating side tube, which latter forms a prolongation of the hollow axle on one side. This is convenient when employing a source of light which does not move with the apparatus as does the Geissler tube, the illuminating centre remaining in the prolongation of the axis of the side tube whatever the tilt of the telescope. The latter may be laid in the bearings with the illuminating tube on either side according to convenience; it is only necessary to remove the two screws which in each case secure the caps of the bearings in order to effect a change of side. The telescope is suspended sufficiently high above the transverse part of the bearing casting to admit of considerable inclination from the horizontal position, and the depression or elevation is effected by a milled-headed screw which passes through a rigid arm radiating from and forming part of the bearing casting, and which presses upwards on rotation against a V-piece attached to the optical tube. Two such V-pieces are provided, on the under and upper sides of the tube. It is convenient to have a means of recording the proper tilt of the telescope corresponding to the adjustment of each of the spectral lines (and therefore of the corresponding signal images and interference

Fig. 5.



bands), whose light is to be employed in the observations. This is delicately achieved by placing a sharp edged flange above the milled head of the tilting screw and employing it as the indicator of a vertical scale, divided on silver to half millimetres, and suspended from the horizontal arm which carries the screw.

The method of suspension is by a slider whose rectangular aperture is slightly larger than the dimensions of the arm at this point, affording means of adjustment by four small adjusting screws driven through the top of the slider, whose points are pulled tightly down upon the arm by two larger milled-headed binding screws passing through the under side of the slider, one on each side of the scale. By means of these screws the scale is readily adjusted to its proper position with respect to the indicating flange edge. The scale is 60 millims. long, this distance corresponding to the traverse by the telescope of rather more than the whole length of the spectrum. Hence a delicate means is afforded of setting the telescope to the proper tilt for the use of either C or F hydrogen light, or that corresponding to the green mercury line, without actually observing the signal image for that colour at every observation. It is only necessary to ascertain once for all, with occasional verifications, the scale readings corresponding to these colours.

The bearing casting of the telescope carries below a vertical conical axis, which rotates within a corresponding hollow cone or vertical bearing, thus admitting of rotation of the telescope in the horizontal plane; the greater portion of this vertical conical bearing passes down into the wide internal boring of the pedestal, and it is rigidly attached to the latter by a screw thread turned on the thickened upper part of its cylindrical exterior and engaging with one in the upper part of the boring of the pedestal. The telescope is prevented from rising in the vertical bearing by a screw driven into the lower end of the solid cone, and a washer broad enough to cover also the end of the bearing. This vertical bearing carries at the top as part of the same casting, immediately above the thread by which it is attached to the pedestal, a broad collar, passing on one side into an arm, intended for use in connection with a fine adjustment fitting placed immediately above it. This latter takes the form of another collar passing round the thickened upper end of the conical axis and continued on two opposite sides into arms. That on one side is short, and through it passes a milled-headed clamping screw to attach the fitting rigidly to the cone, and hence to the telescope; the opposite arm is of the same length as the fixed one carried by the vertical bearing, and its end carries below two elbow pieces, forming two vertical claws, between which the narrower end of the fixed arm passes, with the necessary amount of free play within which the fine adjustment can be effected. The end of the fixed arm is pressed between the fine adjustment screw passing through one claw, and a spring piston carried by the other. Hence, in order to effect fine adjustment for azimuth, it is only needful to tighten the clamping screw, and then to manipulate the fine adjusting screw.

The telescope is provided with a rack and pinion adjustment for the objective, which is of 4 centims. aperture, and is placed at the end of an inner tube, q, which carries the rack, r. This enables the objective to be brought exactly to the focus of the signal aperture of the reflecting prism. There are also provided the means of slightly tilting the objective, in order to throw the troublesome reflection from the first concave lens surface just behind the diaphragm of the micrometer, and hence out of the field of view. This is achieved partly in the mount of the achromatic combination itself, and partly as a final adjustment, by screwing the mount, not to the draw tube itself, but to a cap or short tube, *s*, slipping so easily over the latter that when a pair of screws are passed through both at opposite ends of a horizontal diameter, the cap is free to tilt slightly about them as an axis. By means of another pair of screws at the ends of a vertical diameter, passing through the cap tube only, the latter may be fixed at the slight tilt required. The objective cap is further provided, for use when desirable, with an outer lengthening tube not shown in the figures, projecting outwards 4 centims. beyond the objective, in order to shield it from any extraneous side light.

The reflecting prism, *h*, is cased in metal on the hypotenuse side towards the eye-piece. Between the glass surface facing the objective and the fixed diaphragm there is just room to insert, through a niche cut in the outer tube, any one of the series of interchangeable stops, *k*, suitable guides being provided to prevent actual contact with the glass during the insertion or withdrawal. The stop is in its proper position when it has been pushed home, as far as a handle at the top will permit. The rectangular aperture is placed as near one edge as possible, and when the stop is in position this edge is almost identical with the vertical diameter of the large circular diaphragm aperture, and consequently with the common edge of the semicircular free aperture and the prism; also the centre of the stop lies on the horizontal diameter of the circular aperture. Hence the axis of the bundle of rays incident from the rect-angular signal aperture is as nearly as possible identical with the optical axis, and if the rays reflected from the interference apparatus are brought to a focus in the free aperture, so that the image of the signal is almost in contact with this same vertical diameter, the two paths are practically identical, and the best conditions for inter-ference attained. The apertures of the stops vary from 2 millims. by 1 millim. to 3 by 5 millims., and two series are provided, one with the longest sides of the apertures horizontal, and the others with them vertical; the former are naturally preferable for use with the refracting deflection apparatus of the dilatometer. The smallest size is, of course, preferable, provided the source of light is sufficiently powerful. With the author's Geissler tube the illumination of the bands is many times more powerful for C or F light than it is when using a sodium flame. An additional stop is also provided, which is furnished with a means of varying the vertical height of the aperture, and which is intended for use as a fine slit. At that position in the metallic strip, which will be in the horizontal diameter of the field of the telescope when the stop is in position, a rectangular aperture of 4 millims. width is commenced, and continued upwards as far as and right through the handle. This long aperture runs, of course, parallel to the edges of the strip, and very close to the edge on the side which is to be nearest to the optical axis when in position. Through the handle a movable strip, the same width as the aperture, but longer, so as to project through the handle, passes, adequately guided by miniature guides in the edges of the aperture and corresponding bevels on the slider, the lower horizontal end edge of which forms the upper jaw of the slit. Both this edge and that of the main strip which forms the lower end of the aperture and acts as the lower jaw of the slit, are bevelled. The top of the movable slider projecting above the handle is furnished with a smaller handle, for convenience in setting it so as to adjust the fineness of the slit. This slit enables the spectrum of the illuminating source of light to be studied. As the light traverses the two refracting prisms twice, the apparatus affords a dispersion equal to a spectroscope of four such prisms, and hence the study of the spectrum is highly interesting.

The iris diaphragm, *f*, is placed almost in contact with the fixed diaphragm, *g*; it is manipulated from outside by means of a lever handle traversing a slot, upon the edge of which is a graduated silver arc, the indications of which, corresponding to definite apertures, are recorded by an indicator attached to the handle.

The lens of the illuminating tube is carried in a short sliding tube, *t*, so that the most favourable position for the illumination of the rectangular signal aperture can be attained. When white light is being employed for adjusting purposes, or when sodium light is being used for observations of the bands, this is all that is necessary. When a capillary Geissler tube for longitudinal vision is being employed as source of light, a further arrangement is needful for adjusting the capillary so as to attain the maximum illumination of the signal. The Geissler tube employed is of the H type recommended by PULFRICH, which consists of two wide terminal tubes, *u*, arranged vertically and fitted with aluminium spiral terminals, and a horizontal capillary connecting tube, *v*, the brilliant glow in which, regarded end on, is the source of light. The tubes, after Dr. RIEDEL's pattern, were supplied by ZEISS. They contain a hydrogen vacuum, and a globule of mercury in one of the wide limbs, which is placed furthest from the lens, and on gently warming which the green light due to mercury vapour, corresponding to the green line of the mercury spectrum, makes its appearance. This particular radiation, when separated from the hydrogen radiations by the train of prisms of the expansion apparatus, forms the most effective monochromatic source of illumination for the production of interference bands when the two reflecting surfaces are at a considerable distance.

The supporting and adjusting arrangement for the Geissler tube affords the means of both centering and adjusting, so as to bring the capillary exactly into the prolongation of the optical axis of the illumination tube, and also for moving it longitudinally in this axis to the most suitable distance from the lens. It consists of a tube, *w*, which fits fairly tightly over the broad flange which forms the lens mount of the sliding tube; this flange is of slightly greater diameter than the illuminating tube itself, hence the latter is not injured by the placing in position and removal of this attachment. On the outside of this attached tube slides another, *x*, movable over it by means of a pinion gear carried at the end of the tube nearest the observer and travelling with it, which gears with a rack carried by the inner attached tube. At the outer end of the movable tube is carried a three-disc centering apparatus, y; the second and third discs are complementarily pivoted about points near their periphery, the second about the first, and the third about the second; they are maintained rigidly in position by similar pins on the other side of the centre, working in slots, and their movement is effected by a pair of screws and spring pistons, one carried by the first and another by the third disc, working against projections carried on each side of the middle disc and which are movable in larger slots, cut likewise in the first and third discs. The three discs are pierced by central apertures sufficiently large to admit the rays from the capillary to the lens. Attached to the third disc is a vertical tubular holder, z, for the Geissler tube, one limb of which slides easily in it, a slit being provided in its outer side for the passage of the capillary as far as the centre. Owing to the presence on this limb of some of the author's Geissler tubes of a little sealed side tube, which had been used for the purpose of the exhaustion, the sliding into position of the Geissler tube is brought about from below. When the capillary is raised as far as the slit permits, it is approximately central; a loop support is then attached below it to maintain it in position, by means of a pair of milled-headed screws, which secure the loop to the stout bracket which attaches the tube holder to the third centering disc. The inner side of the tubular holder has a central aperture corresponding to those of the three discs. The bracket is attached in such a manner to the third disc as to provide a means of adjustment for altitude, the claws, one on either side of the holder, not being secured by the pair of screws directly in contact with the disc, but being able to swivel more or less about a horizontal axis formed by a pair of short pins, one carried by each claw between the two screw holes and on the side which would otherwise be in contact with the disc. The slit in which the capillary slides being adequately large, adjustment for azimuth is readily attained. Hence every required adjustment of the capillary is provided for. In order that the Geissler tube may be firmly held, and also that the glass may not be in direct contact with the metal tube, a pair of caoutchouc rings of suitable thickness are introduced as packing, one near the top and one near the lower end of the holder. As the Geissler tube fitting requires to be frequently removed, and it cannot be laid down without the risk of mercury getting into the capillary, an inclined cloth-covered supporting pillar, on a suitable stand, is provided for its reception when not in use. The pillar is of slightly less diameter than the inner tube of the fitting, so as to pass inside it until arrested by the centering discs. The Geissler tube can thus be always left in position in the fitting. It is shown on the side table in fig. 2.

The milled flange, l (fig. 4), which completes the main optical tube, and to the aperture of which the observing eyepiece or micrometer combination is attached, has been constructed in duplicate, one of metal, for use whenever the Geissler tube is not necessary, and another of ebonite, for use in connection with the Geissler tube. For it is found that when the latter is employed, the relatively powerful electrical induction discharge, corresponding to a 5-centim. spark in air, leaks slightly along the metal of the observing apparatus, and if no insulator is interposed, small sparks are discharged from the eyepiece to the observer's eyebrows. The thick ebonite disc acts as a perfect insulator, and prevents this disconcerting occurrence.

The ordinary eyepiece, m, slides in a short tube which is directly screwed to the flange, l, the amount of slide being adequate to enable the observer to sharply focus the vertical edge of the reflecting prism, h, which divides the iris diaphragm into two halves, a semicircle of light (clear aperture) and a dark half covered by the prism.

The micrometer optical combination, n, slides for about 3 centims. in a similar but slightly longer tube attachment to the flange, it is then prevented from sliding further by the thickening of the tube. This thickened part eventually passes into a still thicker one, which is directly attached to the micrometer box, a'; in front of the latter is attached a short tube within which slides the Ramsden double eyepiece, o, which focusses the micrometer spider-lines. Three such eyepieces, of graduated magnifying power, are provided, suitable for all the widths of interference bands likely to be employed. As the movable lens p is only required in two positions, one corresponding to the close approximation of the expansion apparatus and the telescope, and the other to their removal to the two ends of the slate table, two inner sliding tubes are provided, of respectively 15 and 57 millims. length, each carrying at one end an inner thread corresponding with one on the lens mount. When the lens is attached to the shorter tube, and the latter is pushed in the long tube of the combination until the ends of the two tubes are flush with each other, the combination, when in position, focusses the bands for close quarters; when the longer tube, p', is employed, and similarly pushed in until flush, the lens being then in the beginning of the thickest part of the long tube of the combination, the bands are focussed when the expansion apparatus and the telescope are separated at opposite ends of the table. If angle marks are placed on the table as indicators for the position of the corners of the movable slab upon which the expansion apparatus is mounted, corresponding to the sharpest focus under the two conditions, it is easy to bring the slab exactly to one or other of these positions.

The micrometer employed by the author is one in which the spider-lines are moved, not the eyepiece as in the Abbe apparatus. Moreover, it is a double one, carrying a divided drum on each side of the box. Each drum is divided into 100 parts, every 10 being figured. It may be set so that the zero corresponds exactly to any desired position of the spider-line or lines by means of a milled clamping boss carried at the end of the axle, which is here of square section; the aluminium drum itself, being on a round part of the axle, is free to move independently of the spider-lines until the boss is pushed firmly against it and fixed there by a screw driven into the axle, which does not extend so far as to be quite flush with the boss; the drum is then held firmly between the boss on the outside and the thickening of the axle on the inner side. There are two movable vertical spider-lines, and a fixed horizontal one; below and parallel with the latter is the finely-serrated edge of a diaphragm, forming the lower boundary of the field. Every fifth niche of this edge is more deeply indented, and every tenth more still. One of the drums, b', moves one only of the vertical spider-lines, and thus enables it to be set at any recorded distance from the other, or to measure in drum units the distance of any object in the field from the fixed line. The distance between any two teeth or niches of the diaphragm edge is equal to the distance traversed by the spider-line for one complete revolution of the drum. The other, c', moves both vertical spider-lines simultaneously, the movement from one tooth or niche to the next corresponding to one complete revolution of the drum. Hence, each niche corresponds to 100 drum units, and each of the deepest niches to 1000. The micrometer thus affords every convenience for the measurement of the thickness of the bands and the precise location of any one with respect to any point on the horizontal spider-line, and will be equally suitable for any proposed method of procedure. It is particularly convenient for determining the position of a band by bringing it between the two parallel vertical spider-lines, which can be arranged at such a distance apart as is suitable for the purpose.

The fixed point with reference to which the position of the bands at the two limiting temperatures is to be measured, and the number of bands passing which during the change of temperature is to be ascertained, is of the kind adopted by PULFRICH, namely, the centre of a silvered circular spot in the middle of the lower surface of the glass wedge of the interference apparatus. The spot is about 1 millim. in diameter, and the author has removed the central portion so as to leave a silver ring, the inner circular edge of which is employed as the reference circle. Such a ring is readily made by silvering the whole surface by means of a milk-sugar ammoniacal silver solution, and removing the greater portion by ordinary means, and the portion about and within the ring with the aid of a rigidly supported needle and a microscopist's turn-table. The spider-lines are to be placed at such a distance apart as is just slightly smaller than the diameter of the inner silver circle. This enables the centre of the spot to be accurately located by setting the pair of spider-lines so that an equally small arc is cut off from the silver circle on each side. The inner portion is found by the author to be very advantageously left clear, as it enables a band passing the centre to be actually observed at the centre.

To illuminate the Geissler tube, the author employs a 15-centim. spark induction coil, actuated to the extent of a 5-centim. spark. A 100-volt direct supply current is used, filtered through five 32-candle-power lamps arranged in parallel circuit. As each lamp allows 1·2 amperes to pass, the current supplied to the primary coil is 6 amperes, and this current affords a very satisfactory and constant illumination of the tube.

The Thermometers.

The two thermometers, whose lower halves were immersed in the inner bath on each side of the interference chamber, were the two excellent instruments supplied by FUESS, with the most recent form of heating apparatus in connection with his largest gonio-spectrometer No. 1A. They had previously been tested several times, and twice during the course of this expansion work their zero and 100° points were again determined with great care. The zero of both remained exact throughout, and the correction for boiling-point never exceeded 0°·18. Experience has shown, however, that the temperature of the interior of the interference chamber never quite attains the temperature of the inner bath at the higher limits. The difference for the neighbourhood of 70° is about 2°, and for the higher limit of 120° it is rarely less than 4°. These bath thermometers were merely used, therefore, for the purpose of attaining and maintaining a constant temperature in the inner bath. For it is found that if the temperature of the inner bath can be maintained constant to 0°·2, which can readily be attained by the combined use of the graduated gas-tap and thermostat, the actual temperature of the tripod remains constant to well within 0°·1, and as it is capable of exact measurement, and the bands move precisely with the temperature of the tripod, all error of temperature disappears.

The measurement of the actual temperature of the tripod is attained by a third thermometer, specially constructed for the author by Messrs. NEGRETTI and ZAMBRA. This thermometer is bent at right angles just above the bulb, and it is so suspended alongside the interference tube that the small elbow carrying the bulb passes right into the interior of the interference chamber itself, without contact with any part of the walls of the chamber, and the cylindrical bulb rests in actual contact with the upper surface of the platinum-iridium table and one of the screws. Special care has been taken to determine its fixed points from time to time; its capillary was a specially selected one, and its zero and boiling points have required only the minutest corrections. It was so constructed that the 70° mark came just outside the bath, so that no correction for exposed stem is required for the lower limit. For the higher limit of 120° this correction is necessary, and to enable it to be made another much smaller thermometer was attached to the stem, with the bulb opposite to the 90° mark. That the indications of this thermometer actually express the temperature of the tripod and whatever it supports, is proved by the fact that the bands follow it exactly, their motion being arrested simultaneously with that of the mercury column. The importance of actual contact of the bulb of the thermometer with the interference tripod has been very exhaustively proved. Mere hanging of a straight thermometer alongside the chamber, as in the apparatus employed by PULFRICH, affords no guarantee, especially with a closed interference chamber, that the tripod and its contents actually attain the temperature indicated thereby. In the author's experience, even with an open chamber, it never does. The use of oil in the bath cannot do more than slightly reduce the source of error, besides introducing an error due to the impossibility of stirring the oil, on account of the disturbance which a stirrer would cause to such highly delicate observations as are here in question.

Recording Apparatus.

ABBE has shown that it is not necessary to actually follow and count the number of bands which pass the silver spot during the interval between the two limiting temperatures, it being possible to deduce the number by calculation from the data afforded by observations of the initial and final positions of the bands adjacent to the silver spot for light of two different wave-lengths. It is necessary, however, that these initial and final positions should be determined with the utmost precision. In addition to employing this method, which has been elaborately worked out by PULFRICH (*loc. cit.*), the author prefers to remove the slightest possibility of doubt as to the number of bands which pass the reference spot by actually following and counting them, as a mistake of a single band is highly important when the observations relate to the very slight differences of expansion between the members of a series of isomorphous salts. Moreover, it is far more satisfactory to observe the transit of the bands throughout, in order to be assured that it has been unbrokenly uniform, and that there has consequently been no derangement of the adjustment, or cracking, or irregular expansion of the crystal. In order that no error of counting may occur, a method has been adopted by means of which a permanent record of the passage of each band is obtained. A small recording apparatus has been constructed, of such a nature that, by pressing down a key with the finger the moment each band passes the silver spot, a puncture is made in a paper tape, and on releasing the key the tape is moved on a short space ready for the next puncture. When the bands become stationary again, on the attainment of the higher limit of temperature, the tape is cut off behind the last puncture, and thus a permanent record of the number of bands which have made their transit is obtained, which can be counted at leisure, and verified as frequently as may be desired.

The recorder is shown to the right on the accessory table in fig. 2. A drum, carrying a roll of Morse tape, is suspended on an axle inside a rectangular box, the front side of which is hinged below in order that the box may be thrown open, for the purpose of replenishing the drum. For this latter purpose the axle is fixed, and the drum provided with a corresponding central bore; the drum can consequently be drawn forwards right off the axle, and replenished, and is again exactly in position when pushed on the axle as far as it will go, a stout boss attached to a disc screwed to the back of the box arresting its further progress. In order to prevent the drum coming forwards more or less off the axle during working, which would pull the tape out of position, a spring is fixed to the inside of the front lid, and when the latter is closed and is fastened by the two hooks and pins provided for the purpose, this spring presses lightly against the drum and maintains it in position. The tape comes up through a slot in the top of the box, and passes over a small roller raised on supporting bearings about half an inch above the surface. From this roller the tape passes horizontally through the punch and thence between a pair of caoutchouc rollers, which compress it sufficiently to be able to determine its movement when the lower driving roller is rotated. The upper roller is free to rotate on its axle simultaneously with the lower one, but in the opposite direction, by simple contact with the latter; its axle is adjustable for height in order to obtain the necessary amount of pressure between the two rollers, by means of two little milled-headed screws carried on the top of these supports; the screws press down upon rectangular sliders carrying the axle and sliding in slots cut in the upright supports. The axle of the lower roller is continued backwards just beyond the back of the box, where it carries, rigidly attached to it, a ratchet wheel. The movement of the latter is determined by a click, maintained in position against the teeth by a spring; the click is pivoted to and in front of a short lever, the fulcrum of which is the end of the roller axle, upon which it freely turns. The teeth of the ratchet are so arranged that upward movement of the lever and click causes the latter merely to pass round the tooth, while downward movement causes the click to force the ratchet round by one tooth. The lever is connected with a longer one by a short vertical link; the longer one is pivoted at the end of a prolongation of the drum axle, and its movement is directed by another vertical link, which is hinged to the back end of the punch lever. This latter is supported by two fulcrum upright supports, which form the bearings of the horizontal axle about which it is capable of movement. The longer front end terminates in a knob above and a cylindrical rod beneath; the latter passes slightly through a hole in the top of the box when the lever end is up at its normal height. The lever is kept up by a strong spiral spring wound round the rod and slightly compressed between the lever end and the top of the box. Between the knob and the fulcrum the lever carries below a thickening, directly over the punch head, not in contact with the latter, but separated from it by about 4 millims. The small cylindrical punch is provided at the top with a circular head carrying an annular groove, in which gear the two prongs of the forked end of a strong strip of steel spring. The spring is fastened at its other end to a suitable raised support, and the punch passes down, but not quite so far as the tape, through a guiding block provided with a corresponding vertical bore and fixed to the upper plate of the punch bed. Underneath this, leaving a space sufficiently thick for the passage of the tape, is the steel plate of the punch bed, whose sharp-edged circular aperture corresponds to the punch, which fits it sufficiently tightly. It will be at once evident that on pressing down the knob of the lever and thus compressing the spiral spring round the rod, after traversing the 4 millims. space the punch will be pressed through the tape, and the click of the ratchet will be slipped upwards round one tooth; on releasing the knob, the spring pushes it upwards again, but the click now drives the ratchet round for the space of the tooth, thus causing the caoutchouc rollers to draw the tape onwards for a corresponding space. The size of the ratchet teeth is so arranged, relatively to the amount of movement given to the click, that the movement of the ratchet and the tape does not occur until the punch has been extracted from the tape.

Reading Telescope.

A further accessory is a small reading telescope, by means of which the thermometers can be watched by the observer without leaving his seat. This is mounted upon a standard provided with the usual means of adjustment for height, altitude, and azimuth. An inner sliding rod, capable of being arrested at any height by means of a clamping screw passing through the upper boss of the fixed standard, affords the means of adjusting the height of the telescope. Azimuth is arranged for by attaching the telescope to a short tube sliding over the lengthening rod, and fixable at any position by means of a slit and tightening collar carried at one end of the tube. Altitude is provided for by the means of attachment to the sliding tube just referred to, which carries a short projecting horizontal rod over which slips tightly a corresponding short, narrow, horizontal tube, carried by the wider collared tube through which the telescope is inserted; rotation of the narrow tube over the rod enables any desired tilt to be given to the telescope, or the latter to be arranged precisely horizontally with the aid of a spirit level laid along the optical tube. A further adjustment is attainable by means of a joint in the lengthening rod, which enables the telescope to be brought to the most convenient position for the eye when it is impossible to do so by moving the standard bodily. A strong clamping screw is provided with the joint. At the summit of the lengthening rod is carried a clamp which supports a large circular screen of dark-green silk, which serves the double purpose of protecting the observer's eyes from the glare of the vacuum tube, and interposing a broad non-conducting surface between him and the electrical connections of the tube. The whole arrangement is seen in position in fig. 2 (p. 318).

Accessory Interference Apparatus for Observing the Fizeau Phenomenon.

The "Fizeau phenomenon" of periodical secondary interference, produced when light not strictly homogeneous is employed for the generation of the bands, may be very conveniently observed with the aid of the little accessory to the interference apparatus which is shown resting on the movable slab of the expansion apparatus in fig. 2, and which enables the separation of the two reflecting surfaces to be varied at pleasure.

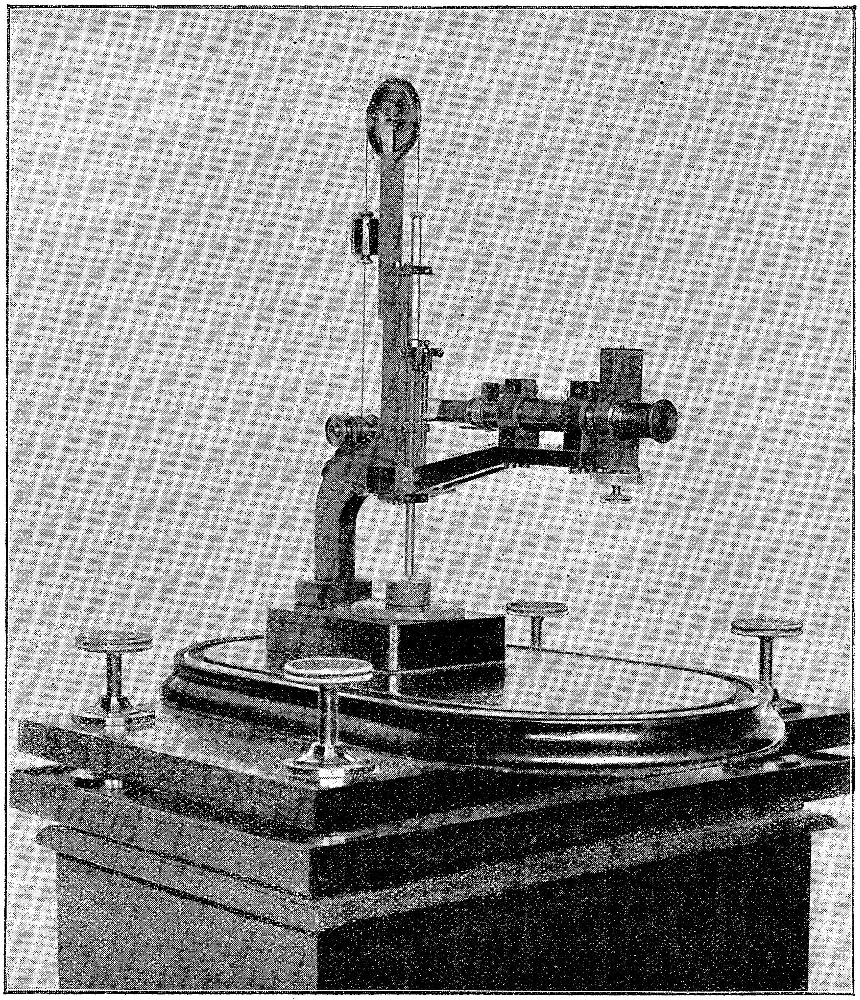
It consists of a special base similar to the ordinary one (k of fig. 3), and adjustable table (g of fig. 3); the latter carries three adjustable screws to act as an interference tripod for the support of the large glass cover-wedge, and a platform, adjustable for height, for the support of the object whose upper horizontal surface is to act as the lower reflecting surface. The base has a much larger central aperture than the ordinary one (k of fig. 3) in order to permit the passage of a stout, hollow cylinder, rigidly carried below by the table; otherwise it is similar to the ordinary one, having three screws passing through it from beneath, pressing upwards against the table for the purpose of adjusting the latter, which is pulled firmly down upon them by a stout spiral spring, confined between the under side of the base, and a collar carried by the hollow cylinder. Within this hollow cylinder slides another, also very stout walled, and closed at the upper end, which is truly plane perpendicular to the axis and forms the vertically movable platform. The fit of the two cylinders is a very close one, and the movement is effected by a pinion, whose manipulating milled head gears in a rack sunk in the movable internal cylinder. It is imperative that there should be absolutely no alteration of the parallelism of the vertical axis during movement, as the minutest amount of such alteration is sufficient to entirely alter the position and width of the bands; hence the closeness of the fit and the relatively considerable length of the cylinders. In order that the amount of separation of the two surfaces may be accurately determined, the milled head of the pinion carries at its inner side a divided drum, reading directly to quarter millimetres with the aid of an indicator carried by the outer cylinder. The three tripod screws are adjustable for height by manipulation of the milled flanges carried some little distance below the pointed upper ends, which enable the screws to be more or less screwed into the fixed nuts carried on the surface of the table. The base carries a screw thread on its periphery, similar to that on the ordinary one, for attachment by means of the tapped flange to the lower end of the interference chamber.

Thickness Measurer.

The measurements of the thickness of the objects investigated, and of the various accessory aluminium blocks and glass discs employed in the work, are carried out by the author with the aid of an admirably accurate thickness measurer of the same type as that furnished by ZEISS, for use in connection with the Abbe dilatometer, and described by PULFRICH in the 'Zeitschrift für Instrumentenkunde,' 1892, p. 307. An improvement is introduced, however, into the method of suspending the counterpoised vertical silver scale, which does not form the continuation of the agate-pointed contact rod, but is suspended in front of the latter, in such a manner as to allow of a delicate means of adjusting the zero of the scale exactly to the centre of the pair of parallel horizontal spider-lines of the micrometer eyepiece of the microscope, when the agate point is resting upon the glass disc fixed in the base.

The instrument is shown in fig. 6, from which the nature of the improved method of suspension will be apparent. Round the rod, at the suitable height, a collar is fixed by a clamping screw; the collar is continued forwards and sideways into a bracket, between the two arms of which the scale is indirectly suspended. The upper end of the scale slides tightly in a short jacket which is directly suspended from the arms of the bracket by two screw-pins. The jacket is pierced in front by a square aperture, through which passes a projecting piece, slightly less in size, screwed firmly to the scale; above and below the aperture the jacket carries two adjusting screws between which the projecting piece is firmly gripped, and which afford the desired means of adjustment.

Fig. 6.



A sliding piece, fixed to the scale half way down, but sliding round the rod, maintains the scale strictly parallel to the rod. The mode of counterpoising the latter so that the agate point may not exert undue pressure upon the object, is the same as in the Zeiss instrument. The scale is divided directly to fifths of millimetres, each division being consequently 0·2 millim. The drum of the micrometer is divided into 100 parts, the tens being figured as units. For one complete revolution the pair of horizontal spider-lines travel vertically through half a scale division, that is, 0·1 millim

The tenths of millimetres are also indicated by the teeth of a serrated diaphragm, which forms the left edge of the field of the micrometer eyepiece, and whose centre, corresponding to the zero of the scale, is specially indented. The figures on the drum thus represent the second place of decimals of millimetres; and the small divisions of the drum represent the third place. Hence the instrument reads directly to 0·001 millim.

Adjustment and Use of the Apparatus, and Determination of its Constants.

Before the apparatus can be employed for the determination of the coefficient of the absolute expansion of any substance, it is necessary to determine the expansion of the platinum-iridium alloy of which the interference tripod is constructed, in order that the expansion of the three screws of the tripod may be known. Having ascertained this, the expansion of any substance may be determined by Fizeau's method, provided a homogeneous specimen of the substance can be obtained in the form of a stout block, furnished with two parallel surfaces at least a centimetre apart. For the author's purpose of being able to determine the expansion of a substance with a block only half a centimetre thick, it is essential to proceed immediately with the determination of the expansion of the specimen of aluminium from which the series of compensators are cut. A block between 12 and 13 millims. thick was employed for this purpose. If the method is adopted of employing the compensator above the crystal, these are all the data required before being able to employ the apparatus directly for the determination of the absolute expansion of a crystal or other small object. If the compensator is employed below the crystal, a determination of the expansion of the glass of the small plates, to be used as covering plates with crystals whose surfaces take a duller polish than glass, also requires to be carried out. A block of the glass, 13 millims. thick, was similarly employed for this purpose. The operations of making these preliminary determinations of the constants of the interference apparatus will so fully include all the necessary manipulative and adjusting operations in connection with the use of the apparatus, that the description of the mode of conducting them, and the discussion of the mode of calculating the coefficients of expansion from the experimental results obtained, will at once be proceeded with.

Determination of the Expansion of the Platinum-Iridium Interference Tripod.

This is the most difficult of all the operations in connection with the work, on account of the fact that it is necessarily performed with the two reflecting surfaces, the under surface of the large glass cover-wedge laid upon the three screws and the upper surface of the platinum-iridium table, separated at the relatively long distance of somewhat over a centimetre. At this distance it is most difficult to generate satisfactory bands with C or F hydrogen light, and it is only at all possible with sodium light provided the exact distance of a FIZEAU maximum is attained. The use of the light corresponding to the green mercury line, as recommended by PULFRICH, affords, fortunately however, excellent bands in the author's apparatus, even at distances considerably beyond 2 centims. At 12 millims., between glass surfaces, the bands are really admirable. A further difficulty, however, arose, owing to the greater volume of reflection from the polished plane surface of the platinum-iridium table than from the glass cover-wedge, the excess of light from the former illuminating the dark spaces and causing the bands to appear very faint. This difficulty was overcome by depositing upon the relevant glass surface, by means of a milk-sugar ammoniacal silver solution, a thin film of silver of the right density to afford the necessary increased reflection from the glass and diminished reflection from the platinum-iridium. After numerous attempts a film was obtained which caused the reflections to be almost exactly equal, and consequently enabled excellent bands to be obtained. The reference mark of the disc was a minute central ring, where the silver had been scraped off by a needle point whilst rotating the disc on a microscopist's turn-table. It should be remarked that this cover-wedge, employed for the purpose of the determination of the expansion of the tripod, was a duplicate of the ordinary one carrying the silver ring previously referred to. A corresponding duplicate smaller disc was employed in connection with it to close the upper aperture of the interference chamber, both being cut from the same slightly wedge-shaped slab of glass, as in the case of the ordinary pair. The two pairs, being of slightly different angle, were provided with distinguishing marks on their edges to prevent the use of a wrong combination.

The screws of the tripod were adjusted so that about 12 millims. length projected beyond that side of the table which carried no points, and the tripod was placed in the interference chamber with this side uppermost. The glass floor of the chamber had previously been made exactly horizontal with the aid of a circular spirit level and the adjusting screws. In order to prevent reflection of light from the marginal portion of the platinum-iridium surface, a lens stop of 15 millims. aperture and convenient diameter was laid upon it as a mask. The cover-wedge was then laid upon the screws with the silvered side downwards, and the engraved chord, marking the direction of the wedge, arranged perpendicular to an imaginary line at this height joining the pedestals of the expansion and the observing apparatus, that is, running right and left of the observer in front of the latter. The counteracting upper wedge was arranged with its chord in a parallel direction and on the same side of the centre to that of the cover-wedge, but below, while that of the cover-wedge was uppermost, on the non-silvered side.

The expansion apparatus was arranged close up to the observing apparatus for the adjustment, and carried the large single reflecting prism in position at its summit.

The heating bath had been temporarily removed. The telescope was fitted with the simple eyepiece and was arranged for parallel rays, the eyepiece focussing the vertical line of division of the iris aperture formed by the edge of the little reflecting prism. An incandescent electric lamp with ground glass globe, and provided with an opaque shade pierced by a circular aperture the same size as the illuminating lens, was brought close to the end of the side illuminating tube, the Geissler tube fitting having been removed. The telescope was then directed at the reflecting prism of the expansion apparatus, and the prism of the latter adjusted so that the images of the rectangular signal stop in front of the small reflecting prism of the telescope, reflected from the four glass surfaces of the two wedges and from the platinum-iridium table, could be viewed and any one brought into the centre of the clear half of the iris aperture by slight movement of the telescope. The iris aperture should be open to the full during this operation. By use of the rack and pinion of the telescope any slight want of sharpness of the images can immediately be corrected.

There are, in general, five such images from the interference chamber, two due to the two surfaces of the upper wedge, two to the cover-wedge, and one from the object, in this case the platinum-iridium table. They are easily recognisable. By slightly lifting and replacing the cover-wedge, the two due to that are distinguished by their temporary movement or disappearance. The angle of the wedge is such that the two images are so far apart in a horizontal line as to be just incapable of being simultaneously visible in the semicircular field of view. The pair of images due to the upper correcting wedge are equally distant from each other, but they should be about the same distance below the other pair as they are apart from each other, and a little to one side, owing to the suitable tilt given to the wedge by the two small tilting screws. These four images are thus arranged at the four corners of a rhombus.

The first thing is to distinguish which of the two cover-wedge images is afforded by the lower surface. The author always works with the thickest side of the wedge to the right, and this causes the image from the lower surface to appear to the right of the other. Hence, the image in question forms the right top corner of the rhombus. Moreover, if the lengths of the platinum-iridium screws had been adjusted to approximate equality, the image due to the object or tripod table would lie very near to this. The next thing is to adjust the image from the lower surface of the cover-wedge so as to lie symmetrically to the horizontal diameter of the field, and almost in contact with the vertical edge which divides the light half from the dark half of the field. This is achieved by movement of the telescope by the fine azimuth adjustment screw and the tilting screw. The next operation is to bring the object image exactly to the same height as the one just adjusted, that is, also symmetrical to the horizontal diameter. If this is not done exactly, the interference bands will be oblique to the vertical diameter of the field instead of parallel thereto, as is desired. To achieve it, one of the screws of the tripod is manipulated, and if the tripod has been conveniently placed so that one screw is to the left side, in the diameter of the chamber floor at right angles to the imaginary line at this height joining the pedestals of the expansion apparatus and the telescope, the screw to rotate may be either the front or back one, at pleasure. It then only remains to manipulate the screw to the left, in order to bring the object image so as to more or less cover the adjusted image, according to the size of signal stop employed and the width of interference band desired. If the two images were absolutely coincident, the conditions would be those for the production of alternate fields of light and darkness, rather than bands, during alteration of temperature and the resultant alteration of the thickness of the air space between the two reflecting surfaces, which, under these conditions, would be precisely parallel. As parallelism is more and more deviated from, by manipulating the left screw so as to bring the object image to cover less and less of the cover-wedge image, the conditions are those for the production of bands, at first few and very wide, but becoming narrower and narrower, and proportionately more numerous, as the process of uncovering the first image proceeds. If an object were supported on the tripod, and the air wedge consequently thin, the bands would be immediately visible if the iris diaphragm were so far closed as to exclude all images but the double one, the simple eyepiece replaced by the micrometer combination, and the white source of light exchanged for a sodium flame.

In order to employ mercury light for the determination of the expansion of the screws of the tripod, or as in ordinary determinations, red or greenish blue hydrogen light, the single reflecting prism of the expansion apparatus is replaced by the pair of refracting prisms. These have previously been set for minimum deviation for the green mercury line, which occupies a convenient mean position in the spectrum, with the aid of the special slit signal stop. The source of white light is removed, and the Geissler tube and its fitting attached, and connected to the induction coil. The telescope is raised to the height of the upper refracting prism, and the induction coil set in action. Instead of the four simple images, the right top one a double one, there are now visible four corresponding vertical spectra, each showing sharp brilliant images of the signal stop in the red and greenish blue, corresponding to C and F hydrogen light, and a fainter one, so long as the tube is at the ordinary temperature, in the bright green, corresponding to the green mercury line. The reason for arranging the white light images as a rhombus, rather than as a square, is now apparent; for if the double image in white light had another image vertically below it in the field, in the Geissler tube light the spectrum of the lower image would partially overlap the double one due to the surfaces whose reflections are to produce bands, and the interference bands would be blurred by the illumination due to the undesired radiations. The angle of the rhombus is such that when the iris is arranged so as just to permit of the complete passage of the desired double image in any of the three colours, the other spectrum is entirely excluded.

The telescope is now to be so arranged for height, tilt, and azimuth that the double image in the desired colour is symmetrical to the horizontal diameter of the field, and almost in contact with the vertical line of demarcation between the light and dark halves of the field. The paths of the incident and reflected light are then almost identical, and the condition for maximum brilliancy of bands is attained. The micrometer combination, fitted with the movable lens carried in its shorter tube, is now attached in place of the simple eyepiece, and its eyepiece is withdrawn, to enable the observer to inspect the double image after the change and ascertain that it is still in position. For it conveniently happens that when the eyepiece of the micrometer is withdrawn, the movable lens left in position acts as a simple eyepiece, although not quite so well as the simple eyepiece supplied for the purpose, being further from the eye and not quite at the focus of the vertical edge of the semi-circular field; it affords, however, images which are sufficiently good for the purpose of adjustment of their proper position with respect to the iris aperture, or of the relative position of the two overlapping images required to produce the bands.

On replacing the eyepiece in front of the micrometer, the interference bands are at once seen in the colour adjusted for. If an object is being used, and the air wedge is thin, the bands are exceedingly brilliant in red hydrogen C light, and only slightly less so in greenish blue F hydrogen light. In green mercury light they are faint at the ordinary temperature of the Geissler tube, but most brilliant if the capillary and the limb containing the mercury globule are gently warmed. The observer no longer sees a semicircular field, as with the simple eyepiece, but a full circular field, completely covered with bands if the object is as large as the diaphragm containing the upper glass wedge of the interference chamber. The semicircular opening, which is really very small when the iris is arranged as has been described, now simply acts as a diaphragm to cut off undesired light. The disturbing reflection from the inner surface of the objective, which would otherwise be very serious, is also thrown behind either the iris diaphragm, or the micrometer serrated diaphragm, by the device for slight tilting of the objective which has been described in the earlier portion of the memoir (p. 328).

The bands can be adjusted to the desired width by manipulation of the left-hand screw of the tripod, and any inclination from the vertical, as observed by reference to the pair of vertical spider-lines, can be corrected by slight rotation of the tripod, or, if preferred, of the micrometer itself within its outer carrying tube. The most suitable width is dependent upon the nature of the object. If its reflecting surface is truly plane, as proved by the severe test imposed by its use for the purpose under discussion, namely, by its generation of rectilinear bands at regular distances, the width of band chosen may be more considerable than when the bands are slightly curved or the distance is not quite regular, as the irregularities largely disappear when the bands are closer together, produced by an air wedge of greater angle. Usually a breadth of about 100 drum divisions, one complete rotation, is a convenient one. The two vertical spider-lines are then to be arranged at such a mutual distance as best facilitates the setting of a band to their centre, and if the width mentioned is chosen, the spider-lines are then also at the most suitable distance apart to enable the centre of the reference mark—the inner edge of the silver ring of the ordinary cover-wedge, or of the clear ring of the silvered cover-wedge used in the observations of the tripod expansion—to be located; in either case the desirable small segment of the circle is visible outside each spider-line when the ring is set for a drum-reading of its position. The horizontal spider-line is to be arranged as a tangent to the ring at the lower end of its vertical diameter. This is achieved by use of the rack and pinion of the pedestal.

The surface of the author's platinum-iridium tripod, truly plane to all ordinary tests, such as reflection of goniometer signals, proves to be very slightly concave, the bands in green mercury light being slightly curved, that is, arcs of large circles. They are perfectly regular, however, and move parallel (concentrically) to themselves when the temperature is altered, demonstrating the fortunate perfect equality of expansion of the whole of the parts of the tripod, and particularly of the screws, and enabling excellent determinations of the expansion to be made by observations of the number of bands passing the centre of the reference circle. A width of band of about 60 drum divisions was found most suitable in these determinations of the expansion of the tripod itself.

The preliminary adjustment having been carried out, the screws of the tripod are rigidly fixed by tightening the little clamping screws. The bent thermometer is then to be carefully attached. It is held near its upper end by a pair of loops at the ends of a circular brass spring collar, fitting just under the flange at the base of the prisms. This mode of support admits of sufficient play to enable the half of the lid, through a hole in which the thermometer is to pass, to be slid over the bent lower end and up the stem into position, and the half lid is supported by the upper end of the interference chamber while the bulb of the thermometer is carefully arranged in proper contact with the upper surface of the tripod table and one of the screws; the bulb may then be secured in this position by means of silk thread. After ascertaining that this operation has not impaired the adjustment, or correcting any slight derangement, the expansion apparatus is smoothly removed, by means of the handles of the cloth-lined slab upon which it is mounted, to its marked place at the further end of the slate table. The micrometer is detached, and its movable lens extracted, unscrewed from its short tube, and fitted to its longer tube, which is then pushed home in the tube of the micrometer until the ends are flush; the micrometer is then placed in position, being now in a condition to sharply focus the bands and reference mark for the new position of the expansion apparatus. The height of the telescope will require slight alteration, and a little further adjustment for azimuth may be necessary; removal of the front eyepiece will, as before, enable the double image to be properly adjusted. The height adjustment is best achieved while the eyepiece is in position and the bands and reference circle in view, the rack and pinion of the pedestal being manipulated so as to again bring the lower margin of the little circle into apparent contact with the horizontal spider-line.

The heating bath may now be placed in position upon its annular support, which latter is always left *in situ*. To enable this to be done, the interference tube is gently raised, by the rigidly moving rack and pinion, to the necessary height, indicated by a mark on the sliding column. A large circular sheet of asbestos millboard, pierced by the necessary apertures, is intended to be laid on the top, and a large semi-cylindrical screen of the same material is placed between the bath and the pedestal for the full height of the latter. It is most convenient that the circular sheet should be arranged while fixing the bent thermometer, above the two halves of the lid, which are readily supported on the upper end of the interference chamber. Several circular sheets of asbestos are already laid beneath the Fletcher ring gas-burner, alternated with porcelain tiles. The interference tube is then lowered into the bath, another mark on the sliding column indicating the proper position, which, as adjustment had designedly been made with the column at this same height as indicated by the mark, will be the same as before. The two halves of the lid and the asbestos circle are left in position by the sinking of the interference tube, and the wide aperture necessarily left in the asbestos for the passage of the interference chamber is covered with two additional overlapping semicircles of asbestos. The thermostat and the bath thermometers may next be placed in position and the gas connections made. In order to protect the dispersing apparatus as much as possible during the heating a further circular screen of mica, arranged in two halves, attached by studs at each end and perforated by suitable apertures at the overlapping inner edges, is fixed about the height of the top of the porcelain tube. It is conveniently supported by the top of the semi-cylindrical screen and a couple of asbestos supports laid on each side of the front part of the asbestos circle, leaving ample space between for a clear view of the thermometers. By this means the temperature at the dispersion apparatus is only appreciably higher than that of the room when the thermometers are indicating the highest limit of 120°.

It should be here stated that the author was much troubled at first by the absorption or condensation of moisture in or on the copper walls of the inner bath. It is quite unnoticeable at the ordinary temperature, but when the temperature is raised and the transit of bands commences, the latter become suddenly obscured and the observation is lost, owing to the condensation of a film of distilled moisture on the glass wedges in the upper part of the interference chamber. This awkward circumstance necessitates the abandonment of the work for the day, as the apparatus requires to stand at least twelve hours to re-take the atmospheric temperature. It can be completely avoided, however, by placing on the floor of the bath, after each determination, so as to desiccate it before the next, a shallow dish of vitriol, and

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leaving the interference tube immersed in the bath, so as also to share in the desiccation. It is a great advantage if the adjustment can be carried out the evening before a determination, as not only is there then ample time for the temperature of the room and apparatus to attain equilibrium, but also vitriol can then be left in the bath during the night, and removed just before the observations commence in the morning. For if the bath and interference apparatus are long exposed to moist air during adjusting operations, and the tripod subjected to much manipulation with the fingers, condensation again occurs to a greater or less extent which it is desirable to remove. The fixing of the mica screen should be left till this is carried out. The momentary raising of the interference apparatus from the bath in order to remove the vitriol dish serves, moreover, to enable the observer to note the reading of the inner bent thermometer, whose bulb is in contact with the tripod. This temperature is, of course, taken as the starting temperature in preference to that indicated by the bath thermometers, in case any slight difference is apparent.

Finally, before commencing actual observations, the bands are closely examined, and any slight alteration of the height of the reference ring above the horizontal spider-line corrected; also the eyepiece should be momentarily removed to satisfy oneself that the double image is still correctly placed. The observation of the position of the bands for the starting temperature can then be proceeded with.

The method adopted for determining the position of the bands, with reference to the centre of the silver ring, is that recommended by PULFRICH. Two things require to be ascertained. First, the distance between the centres of two adjacent dark bands, which may be termed the width of the bands; and second, the distance between the centre of the nearest dark band and the centre of the reference ring. The quotient of the second by the first is evidently the fraction of a band by which the centre of reference is distant from the nearest band. When the temperature is raised, and the bands move past the vertical spider-lines, the first dark band which comes between the pair of spider-lines is to be counted, not as one band, but as the fraction of a band thus determined, in case the movement of the nearest band occurs in the direction of the centre; and in the contrary case as the complementary fraction, that is, unity minus the determined fraction. The whole observation is completed by counting or otherwise determining the number of succeeding dark bands which pass during the interval until the temperature again becomes constant at the higher limit, and by then determining in a similar manner as for the lower limit the fraction of a band to be added, beyond the last which has passed the centre. Hence, the essentials of the determination are the whole number of bands which pass, and the initial and final fractions of a band to be added to this whole number; and the determination of each of these fractions involves the two operations previously referred to.

To determine the width of the bands the author measures the five whose middle one is immediately to the left of the centre of the little reference ring. Duplicate readings of the micrometer drum, the one which moves both spider-lines simultaneously, are taken, corresponding to the setting of each of these five bands in turn, precisely midway between the two spider-lines. It is convenient to call these values a, b, c, d, e, letting a be the one most to the left; the values will then be in ascending order of drum divisions, each complete revolution of the drum being 100. In addition to the four direct values for the width of a band obtained by taking the differences between successive readings, a considerable number of other combinations can be obtained for the width from the five readings, and PULFRICH has shown that the whole of the values reduce eventually, converting his symbols into the author's, to the simple formula:

$$w = \tfrac{1}{6}(d + e) - (a + b).$$

To determine the distance of the middle band, c, from the reference point, it is only necessary to determine the true position of c by taking the arithmetical mean of the five readings, and to determine the drum reading of the reference point by taking the mean of two or three independent readings corresponding to the adjustment of the silver ring so that equal small arcs are visible outside each of the vertical spider-lines. The difference between these mean readings for the band c and the centre of the reference ring is evidently the distance δ required. The fraction of a band corresponding to this temperature is, then, either δ/w or $\dfrac{w - \delta}{w}$, according to the direction in which the bands are found to move on raising the temperature.

The Fletcher ring gas-burner is then ignited, the gas supply to it being regulated by the graduated lever tap, so that the temperature of the bath may not rise too rapidly. As has already been explained, the author prefers to actually observe the passage of the bands, and to count their number with the aid of the recording apparatus, rather than to rely exclusively upon the ABBE method of calculation from observation with two wave-lengths. Moreover, in the case of the determination of the expansion of the tripod, the bands are so very much clearer with green mercury light than with either C or F hydrogen light, or sodium light, at the necessarily long distance apart of the two reflecting surfaces, that the author prefers to base his determinations entirely upon observations in green mercury light. The recorder is placed upon the little cloth-covered accessory table to the observer's right, and as each dark band (after the first, which has the determined fractional value) advances to the middle of the two vertical spider-lines, the knob is pressed and the puncture made in the tape.

For a reason which will be fully discussed in considering the mode of calculation of the experimental results, the author makes two series of observations, the upper temperature limits of which are in the neighbourhood respectively of 70° and 120°.

The rise of temperature is so regulated, that fully an hour is occupied in attaining the neighbourhood of the first higher limit, about 70°, the transit of the bands occurring with more or less regularity during this interval. A further valuable control of the number of bands passing the spider-lines is afforded if the observer notes the passage of each quarter of a band in his note-book. The author calls the quarters respectively 1, 2, 3, 4, and as the four successively make their transit, puts down their number on the same line; after 4 is so put down, the puncture is immediately made on the recorder, and a "p" added after the 4 in the note-book, to signify that the puncture has been made. If, also, the number of this particular band is added, an additional check is obtained. It may be mentioned that, in the case of the tripod determination, and all other cases where the bands are not so numerous as to pass with great rapidity, it is not necessary to keep the induction coil in action during the whole time; it is sufficient to run it intermittently in such a manner as to observe the passage of the four quarters of each band. This prevents overheating of the Geissler tube, and considerably prolongs its efficiency. The hydrogen spectrum rapidly deteriorates if the coil is worked continuously. A further period of two to three hours is then allowed for the attainment of equilibrium at this temperature, during the last hour and a half, at least, of which the temperature is maintained constant with the aid of the thermostat and the graduated gas-tap. It is found that if constancy to within $0^{\circ}{\cdot}2$ is attained in the inner bath, as indicated by the outer two thermometers, a degree of constancy fairly easy to maintain, the actual temperature of the interference apparatus, as indicated by the inner bent thermometer whose bulb is in direct contact with the platinum-iridium tripod, remains absolutely constant. For the last ten degrees or so of the rise of temperature, the bands move precisely with the mercury column of this thermometer, and their motion ceases exactly with the attainment of constancy by it, a circumstance which is eminently satisfactory. When the last band to pass the vertical spider-lines for this temperature interval has been recorded, and no further movement of the bands is observable, the tape is drawn forward some distance, in order to interpose an interval between the last puncture for this temperature interval and the first of the next, and a series of measurements similar to those at the starting temperature are made, in order to determine the fraction of a band which has passed since the last transit.

When these measurements are completed, the gas supply is again increased so as to gradually raise the temperature to the highest limit, about 120°. The transit of bands is again observed and recorded, the same interval as before is allowed for the attainment of equilibrium and constancy, and finally, the determination of the last fraction of a band is made in the same manner as the two former determinations. The gas supply can then be turned off, and the apparatus allowed to cool. The tape is again drawn forward, and as the determination is finished, cut off. It is convenient to leave ample tape on each side of the punctures for a brief description of the determination to which it refers; also to add at each end of the row of punctures corresponding to each of the two temperature intervals the values of the fractions to be added. In the case of the first puncture of the second interval, this only represents the complementary fraction to that to be added to the punctures of the first interval, so this fraction is set, by exception, over the top of this particular puncture rather than alongside it. It is convenient to actually make this puncture, because the total number of punctures made for the two sets of observations then represents the actual number of dark bands which have made their transit between the starting temperature and the highest limit.

Three quantities are the result of these observations :

(*a*.) The number of whole bands and fraction of a band which have passed the point of reference between the starting temperature and the first higher limit (about 70°). This is given by the number of punctures for the first interval plus the initial and final fractions for that interval.

(*b*.) The number of whole bands and fraction of a band which have passed the reference point between the first higher limit and the highest limit (about 120°). This is equal to the number of punctures for the second interval minus one, plus the initial and final fractions; or, which is the same thing, the number of punctures plus the final fraction for this interval minus the final fraction for the first interval.

(*c*.) The number of bands and fraction of a band which have made their transit past the reference point between the starting temperature and the highest limit. This is given by the sum of the total number of punctures and the initial and last fractions. Naturally, $c = a + b$.

If we retain FIZEAU'S symbol, f, for the number of bands and fraction of a band passing the reference point during a given interval of temperature, the alteration of thickness of the air film denoted by the observations for that interval is approximately equal to $f\frac{1}{2}\lambda$, where λ is the wave-length of the particular light employed to generate the bands. The wave-length of the green mercury line is 0·0005460 millim. The word approximate is introduced because there is a correction to apply for the alteration of the refractive index of air, and consequent variation of the wave-length involved, due to change of temperature and pressure. The result of this alteration in the wave-length is that the number of bands actually observed to pass the fixed point is slightly different to the number which would have been observed if no change in the wave-length had occurred. This correction is very small, only a small fraction of a band, when the air film is thin, as is usually the case; but it is very considerable, amounting to a tenth of the whole value, when the thickness is relatively great, as it is during the determination of the expansion of the tripod. The nature and extent of this correction have been fully discussed by both BENOIT and PULFRICH (*loc. cit.*), and the formulæ arrived at by each of these observers lead to the same result. After employing both, the author finds the form of expression given by PULFRICH ('Zeits. für Instrumentenkunde,' 1893, p. 456), rather more convenient, as the values of several of the factors can be taken directly from LANDOLT'S, 'Physikalisch-Chemische Tabellen.' The formula is as follows :—

$$f' = f + d\,(t_2 - t_1) \cdot \frac{b_1}{760} \cdot \frac{1}{1 + \alpha t_1} \cdot \frac{1}{1 + \alpha t_2} \cdot 2\,\frac{(n-1)\,\alpha}{\lambda} - d\,(b_2 - b_1) \cdot \frac{1}{1 + \alpha t_2} \cdot 2\,\frac{n-1}{760\lambda},$$

where f' is the corrected number of bands, f the observed number, d the thickness of the air layer, t_1 and t_2 the limiting temperatures, and b_1 and b_2 the corresponding barometric pressures; α is the coefficient of expansion of air 0·00367, and n the refractive index of air for the wave-length λ of the light employed. The logarithmic values of $2\,\frac{(n-1)\,\alpha}{\lambda}$ and $2\,\frac{n-1}{760\lambda}$ can be found once for all; they are respectively $\bar{3}$·59901 and $\bar{3}$·15353. The logarithms of the factors $\frac{b_1}{760}$, $\frac{1}{1 + \alpha t_1}$, and $\frac{1}{1 + \alpha t_2}$ can be extracted directly from LANDOLT's tables. Hence the expression, although apparently long and troublesome, lends itself to very easy computation. For the purpose of the calculation of this correction it is necessary to take the reading of the barometer at the time of making the observation of the exact position of the bands at the initial temperature, and again when they have attained their condition of rest at each of the two higher limits. These barometric observations are of particular importance in the case of the determination of the expansion of the tripod, as the share of the variation of pressure in the correction is then an appreciable one The author employed a standard barometer suspended in the same room.

With regard to the sign of the correction, the signs given in the above formula for the temperature and pressure portions are the correct ones for use in all cases where the result of the increase of temperature is to effect an increase in the separation of the two reflecting surfaces, as in the case in question of the determination of the expansion of the tripod. In such cases the effect of change of temperature is to cause the number of observed bands to be less than it would be if such a change did not occur. The contrary is the case where the thickness of the air-layer diminishes, as in the cases of the determination of the expansion of an object supported on the tripod table whose expansion is greater than that of the screws, and in all determinations of expansion involving the use of the compensator. In such cases the signs of the temperature and pressure portions of the above formula should be respectively $-$ and $+$.

The temperature portion of the correction is usually much the larger, and its sign governs that of the total correction. Indeed, in all cases when the air-layer is very thin, the share of pressure can be neglected. Moreover, as will be evident from the formula, the sign of the pressure portion varies according as b_1 is less or greater than b_2. With regard to the sign of the more important temperature portion, the fact that it follows the rule just indicated will be evident from the following considerations. Rise of temperature causes a diminution in the refractive index of the air, and a corresponding increase in the wave-length. The same amount of separation of the surfaces will, therefore, contain fewer wave-lengths, and if no expansion of the tripod and object (if one is being used) occurred, the observer would perceive the transit of the bands to the extent indicated by the correction, and in the same direction as the bands move when the two surfaces approach each other; so that the effect of rise of temperature on the refraction of the air is similar to that produced by approach of the two surfaces. Hence the correction requires to be added when the effect of rise of temperature is to cause recession of the surfaces, and to be subtracted when the surfaces approach each other.

After the conclusion of the observations of the number of bands which effect their transit for the two intervals of temperature, it is the author's practice to leave the apparatus untouched for about sixteen hours, during which time it regains the ordinary temperature of the air, and subsequently to repeat the whole series of operations and observations for two similar temperature intervals on the following day, and on as many succeeding days as it is desired to make independent determinations. After the completion of the series, and the final slow cooling of the apparatus during the night, the measurement of the exact length of the screws of the tripod above the reflecting surface of the table is made, with the aid of the thickness measurer. In the case of the determination of the expansion of the tripod, this is the only additional quantity required before being able to proceed with the calculation of the results.

The full data now available for this determination are as follows:—

Lt_1, length of platinum-iridium screws at the initial temperature, t_1; in this case $= d$ thickness of air-layer.

t_1, initial temperature; t_2, first higher limit, about 70°; t_3, highest limit, about 120°.

b_1, b_2, b_3, corresponding barometric readings.

f_2, observed number of bands for interval $t_2 - t_1$.

f_3, observed bands for interval $t_3 - t_1$.

By means of the correction formula and these data we first calculate f_2' and f_3', the corrected number of bands for the two intervals. These quantities, when multiplied by $\frac{1}{2}\lambda$, in this case, for green mercury light, 0·000273 millim., afford the amounts of expansion of the screws, or alteration of thickness of the air-layer for the two intervals of temperature; and therefrom, by addition to the measured initial length, Lt_1, we obtain the lengths Lt_2 and Lt_3, at the first and second higher limits (near 70° and 120°) respectively. That is to say, the length of the platinum-iridium screws is now known at three temperatures in the neighbourhood of 10°, 70°, and 120°.

The author's object in making observations for two different temperature intervals is to be able to determine not only the mean coefficient of expansion between two limits of temperature, but also the absolute coefficient of expansion at any given temperature, and the variation of the coefficient with change of temperature. For it is well known that, in general, the linear coefficient of expansion is not a constant quantity but varies slightly with the temperature; the increment of the coefficient per degree, however, remains practically constant. If, therefore, the length of the solid substance at 0° is represented by L_0, and the length at t° by Lt, the nature of the change of length is adequately represented by the formula

$$Lt = L_0 (1 + at + bt^2).$$

If α be the absolute coefficient of linear expansion, then the constant increment per degree is $\Delta\alpha/\Delta t$, and their relations to the constants a and b of the above formula can be ascertained at once by successive differentiation with respect to the temperature.

$$\alpha = a + 2bt, \qquad \Delta\alpha/\Delta t = 2b.$$

The mean coefficient of expansion between 0° and t° is therefore $a + bt$; but the true coefficient at any particular temperature t, and also the mean coefficient between any two temperatures whose mean is t, is $a + 2bt$.

Hence in order to be able to determine the true and mean coefficients, and the increment per degree, which together afford full information as to the nature of the expansion, it is only necessary to ascertain the constants a and b in the general expression above quoted for Lt.

In this formula Lt and t are known, and there are three unknown quantities, L_0, a, and b. To determine them three equations, for three different temperatures, are required. The data derived from the observations at the ordinary temperature, t_1 (about 10°), at the first higher limit, t_2 (about 70°), and at the highest limit, t_3 (about 120°), enable the required three equations to be compiled. For Lt_1 is the length measured at the ordinary temperature by means of the thickness measurer; Lt_2 is $Lt_1 + f_2' \frac{1}{2}\lambda$; and Lt_3 is $Lt_1 + f_3' \frac{1}{2}\lambda$. We have then the three equations

$$Lt_1 = L_0 (1 + at_1 + bt_1^2),$$
$$Lt_2 = L_0 (1 + at_2 + bt_2^2),$$
$$Lt_3 = L_0 (1 + at_3 + bt_3^2).$$

By subtracting respectively the first from the second, and first from the third equations, we obtain a pair of equations which, by complementary multiplication with the coefficients of a and b respectively, enable each of these constants to be eliminated in turn and the other to be obtained in terms only of L_0 and the known quantities. On substituting these values of a and b in any of the three fundamental equations L_0 is at once obtained, and its substitution in the expressions for a and b, just referred to, affords the desired numerical values of these constants.

The three expressions for a, b, and L_0, in the form actually employed by the author in the reductions, are as follows:

$$a = \frac{1}{L_0}\left[\frac{(t_1+t_3)(Lt_2 - Lt_1)}{(t_2-t_1)(t_3-t_2)} - \frac{(t_1+t_2)(Lt_3-Lt_1)}{(t_3-t_1)(t_3-t_2)}\right] = \frac{\theta}{L_0},$$

$$b = \frac{1}{L_0}\left[\frac{Lt_3 - Lt_1}{(t_3-t_1)(t_3-t_2)} - \frac{Lt_2-Lt_1}{(t_2-t_1)(t_3-t_2)}\right] = \frac{\phi}{L_0},$$

$$L_0 = Lt_1 - \theta t_1 - \phi t_1^2.$$

Although the expressions θ and ϕ appear long they are really very readily computed from the experimental data, as there are only three sets of quantities involved, the differences of the temperatures, the sums of the temperatures, and the differences of the lengths at these temperatures; also the two pairs of denominators are inversely identical.

The results of the observations connected with the determination of the expansion of the platinum-iridium tripod will now be given. Five independent series were carried out, in order that this fundamental constant might be ascertained with the utmost precision. With regard to the length of the screws of the tripod, given in the tables of results, it should be remarked that the actual lengths of the three screws are necessarily very slightly different, in order to produce the slightly wedge-shaped air-layer required for the generation of the bands. The length given is of course the mean length, and is the thickness of the air-layer at the centre, where the passage of bands is observed. In order to determine this mean length of the screws, besides direct measurement of each screw, which is somewhat difficult on account of the more or less pointed character of the screw-ends and of the agate end of the rod of the thickness measurer, recourse was had to the use of a plane parallel disc of thick glass, whose thickness was accurately known, and which was laid on the screws.

Expansion of Platinum-iridium Tripod.

Series 1.

Length of screws above table, $Lt_1 = 12{\cdot}369$ millims.

Thickness of air-layer, $d = 12{\cdot}369$ millims.

Temperatures of observations, $t_1 = 11^\circ{\cdot}1$ C., $t_2 = 68^\circ{\cdot}5$, $t_3 = 117^\circ{\cdot}8$.

$t_2 - t_1 = 57{\cdot}4$. $t_3 - t_1 = 106{\cdot}7$.

Barometric pressures, $b_1 = 763{\cdot}9$, $b_2 = 765{\cdot}2$, $b_3 = 766{\cdot}4$ millims.

Number of transited interference bands observed during the two intervals, $f_2 = 20{\cdot}63, f_3 = 39{\cdot}40$.

Correction for alteration of refraction of air, $+ 2{\cdot}17$ and $+ 3{\cdot}52$.

Corrected number of bands, $f_2' = 22{\cdot}80$, $f_3' = 42{\cdot}92$.

Wave-length of light employed (green mercury line), $\lambda = 0{\cdot}0005460$ millim. $\frac{1}{2}\lambda = 0{\cdot}000273$ millim.

Elongation of screws, $Lt_2 - Lt_1 = f_2' \frac{1}{2}\lambda = 0{\cdot}0062243$ millim.
$Lt_3 - Lt_1 = f_3' \frac{1}{2}\lambda = 0{\cdot}011717$ millim.
$\theta = 0{\cdot}000\,106\,22$. $\phi = 0{\cdot}000\,000\,027\,918$.
$L_0 = 12{\cdot}3678$ millims.
$a = 0{\cdot}000\,008\,588\,2$. $b = 0{\cdot}000\,000\,002\,257\,3$.

Series 2.

Length of screws above table, $Lt_1 = 12{\cdot}369$ = thickness of air-layer, d.
Temperatures, $t_1 = 12^\circ{\cdot}0$, $t_2 = 70^\circ{\cdot}0$, $t_3 = 118^\circ{\cdot}5$.
$t_2 - t_1 = 58{\cdot}0$. $t_3 - t_1 = 106{\cdot}5$.
Pressures, $b_1 = 763$, $b_2 = 762{\cdot}1$, $b_3 = 761{\cdot}6$ millims.
Number of transited bands, $f_2 = 20{\cdot}91$, $f_3 = 39{\cdot}44$.
Correction, $+ 2{\cdot}19$, $+ 3{\cdot}52$.
Corrected number of bands, $f_2' = 23{\cdot}10$, $f_3' = 42{\cdot}96$.
Wave-length of light employed, $\lambda = 0{\cdot}000546$. $\frac{1}{2}\lambda = 0{\cdot}000273$.
Elongation of screws, $Lt_2 - Lt_1 = f_2' \frac{1}{2}\lambda = 0{\cdot}0063062$.
$Lt_3 - Lt_1 = f_3' \frac{1}{2}\lambda = 0{\cdot}011728$.
$\theta = 0{\cdot}000\,106\,37$. $\phi = 0{\cdot}000\,000\,028\,739$.
$L_0 = 12{\cdot}3677$.
$a = 0{\cdot}000\,008\,600\,6$. $b = 0{\cdot}000\,000\,002\,323\,7$.

Series 3.

Length of screws above table, $Lt_1 = 12{\cdot}369$ = thickness of air-layer, d.
Temperatures, $t_1 = 12^\circ{\cdot}0$, $t_2 = 70^\circ{\cdot}3$, $t_3 = 122^\circ{\cdot}2$.
$t_2 - t_1 = 58{\cdot}3$. $t_3 - t_1 = 110{\cdot}2$.
Pressures, $b_1 = 761{\cdot}0$, $b_2 = 761{\cdot}2$, $b_3 = 761{\cdot}9$ millims.
Number of transited bands, $f_2 = 21{\cdot}04$, $f_3 = 40{\cdot}97$.
Correction, $+ 2{\cdot}18$, $+ 3{\cdot}57$.
Corrected number of bands, $f_2' = 23{\cdot}22$, $f_3' = 44{\cdot}54$.
Wave-length of light employed, $\lambda = 0{\cdot}000546$. $\frac{1}{2}\lambda = 0{\cdot}000273$.
Elongation of screws, $Lt_2 - Lt_1 = 0{\cdot}006339$. $Lt_3 - Lt_1 = 0{\cdot}012159$.
$\theta = 0{\cdot}000\,106\,18$. $\phi = 0{\cdot}000\,000\,031\,011$.
$L_0 = 12{\cdot}3677$.
$a = 0{\cdot}000\,008\,585\,2$. $b = 0{\cdot}000\,000\,002\,507\,4$.

Series 4.

Length of screws above table, $Lt_1 = 12{\cdot}369$ = thickness of air-layer, d.
Temperatures, $t_1 = 10^\circ{\cdot}8$, $t_2 = 70^\circ{\cdot}7$, $t_3 = 121^\circ{\cdot}1$.

$$t_2 - t_1 = 59{\cdot}9. \quad t_3 - t_1 = 110{\cdot}3.$$

Pressures, $b_1 = 761{\cdot}5$, $b_2 = 761{\cdot}0$, $b_3 = 759{\cdot}7$ millims.
Number of transited bands, $f_2 = 21{\cdot}61$, $f_3 = 40{\cdot}78$.
Correction, $+ 2{\cdot}26$, $+ 3{\cdot}65$.
Corrected number of bands, $f_2' = 23{\cdot}87$, $f_3' = 44{\cdot}43$.
Wave-length of light employed, $\lambda = 0{\cdot}000546$. $\frac{1}{2}\lambda = 0{\cdot}000273$.
Elongation of screws, $Lt_2 - Lt_1 = 0{\cdot}0065164$. $Lt_3 - Lt_1 = 0{\cdot}012129$.
$\theta = 0{\cdot}000\ 106\ 88$. $\phi = 0{\cdot}000\ 000\ 023\ 394$.
$L_0 = 12{\cdot}3678$.
$a = 0{\cdot}000\ 008\ 642\ 1$. $b = 0{\cdot}000\ 000\ 001\ 891\ 5$.

Series 5.

Length of screws above table, $Lt_1 = 12{\cdot}369$ = thickness of air-layer, d.
Temperatures, $t_1 = 10^\circ{\cdot}3$, $t_2 = 68^\circ{\cdot}1$, $t_3 = 118^\circ{\cdot}8$.

$$t_2 - t_1 = 57{\cdot}8. \quad t_3 - t_1 = 108{\cdot}5.$$

Pressures, $b_1 = 765{\cdot}3$, $b_2 = 766{\cdot}0$, $b_3 = 766{\cdot}8$ millims.
Number of transited bands, $f_2 = 20{\cdot}78$, $f_3 = 40{\cdot}14$.
Correction, $+ 2{\cdot}19$, $+ 3{\cdot}58$.
Corrected number of bands, $f_2' = 22{\cdot}97$, $f_3' = 43{\cdot}72$.
Wave-length of light employed, $\lambda = 0{\cdot}000546$. $\frac{1}{2}\lambda = 0{\cdot}000273$.
Elongation of screws, $Lt_2 - Lt_1 = 0{\cdot}0062708$. $Lt_3 - Lt_1 = 0{\cdot}011935$.
$\theta = 0{\cdot}000\ 106\ 15$. $\phi = 0{\cdot}000\ 000\ 029\ 846$.
$L_0 = 12{\cdot}3678$.
$a = 0{\cdot}000\ 008\ 582\ 8$. $b = 0{\cdot}000\ 000\ 002\ 413\ 2$.

The results derived from the five series are compared in the following table, and the mean values for a and b extracted :—

	a.	*b.*
Series 1	0·000 008 588 2	0·000 000 002 257 3
,, 2	8 600 6	2 323 7
,, 3	8 585 2	2 507 4
,, 4	8 642 1	1 891 5
,, 5	8 582 8	2 413 2

Mean values, $a = 0{\cdot}000\,008\,599\,8$, $b = 0{\cdot}000\,000\,002\,278\,6$.

The final result, therefore, for the mean coefficient of expansion $a + bt$ of the platinum-iridium of the tripod, between 0° and $t°$, is

$$0{\cdot}000\,008\,600 + 0{\cdot}000\,000\,002\,28t, \text{ or } 10^{-9}(8600 + 2{\cdot}28t).$$

The true coefficient of expansion at $t°$, or mean coefficient between any two temperatures whose mean is t, $\alpha = a + 2bt$, is

$$\alpha = 0{\cdot}000\,008\,600 + 0{\cdot}000\,000\,004\,56t, \text{ or } = 10^{-9}(8600 + 4{\cdot}56t).$$

It will be interesting to compare the value for the expansion of the specimen of platinum-iridium composing the author's tripod with the values found by BENOIT for various specimens of JOHNSON and MATTHEY 10 per cent. alloys of iridium with platinum. BENOIT's values are as follows; they refer to the mean coefficient between 0° and $t°$:—

Specimen 1, $10^{-9}(8615 + 2{\cdot}21t)$.
,, 2, $10^{-9}(8593 + 2{\cdot}40t)$.
,, 3, $10^{-9}(8598 + 2{\cdot}22t)$.
,, 4, $10^{-9}(8575 + 2{\cdot}38t)$.
,, 5, $10^{-9}(8575 + 2{\cdot}54t)$.

It will be observed that the author's value agrees most satisfactorily with these values of BENOIT. As the coefficient of iridium is much lower, namely, 0·0000065, than that of platinum, 0·0000089, a slight difference in the proportions of the two metals present in the alloy might be expected to produce just such slight variations in the coefficient as are observed.

Determination of the Expansion of the Aluminium Compensator.

The aluminium compensating blocks were cut from the same casting of the purest obtainable aluminium. The thickest block of the 25 millim. series, slightly over 12 millims. thick, was used for the purpose of determining the expansion by the differential Fizeau method. The platinum-iridium tripod screws were preliminarily set to about 12·5 millims., on that side of the table which was furnished with raised points. The aluminium block was placed upon the table, resting on the three outer highest points. The tripod and block were then placed in the interference chamber, and the large glass cover-wedge provided with the minute central silver ring (not the cover-wedge silvered all over, which is used only for the tripod determinations) was placed over the screws. The corresponding counter-balancing wedge had previously been exchanged for the former one in the upper aperture of the chamber, and the engraved direction chords were, as before, arranged parallel, but that of the cover-wedge above and that of the counteracting wedge below.

The adjustment of the upper surface of the aluminium, and of the lower surface of the cover-wedge, was carried out precisely as has been described for the tripod table and cover-wedge; the two parts of the apparatus were arranged at close quarters, and the images of the signal-stop obtained in white light with the aid of the single reflecting prism were brought to the desirable partially overlapping position, and isolated by the iris diaphragm from all other radiations. The single prism was then exchanged for the train of refracting prisms, the white light was replaced by red hydrogen light from the Geissler tube, and the simple eyepiece was replaced by the micrometer combination, when very brilliant bands were at once observed. After adjustment of the width of the bands and their parallelism to the vertical spider-lines, the platinum-iridium screws were fixed, the bent thermometer was arranged in position, and the expansion apparatus removed to its proper distant position. The interference tube was then raised, while the bath, containing its dish of vitriol, was arranged in position; the interference apparatus was then lowered into the bath, the thermometers fixed, and the whole left overnight to attain equilibrium of temperature.

Next morning the vitriol was removed, and the inner thermometer read while the interference apparatus was raised for this purpose; the barometric pressure was also taken, the adjustment of the bands for height effected, the silver ring being brought into contact at its lower limb with the horizontal spider-line, and the measurements of the positions of the centre of the silver ring and of the five adjacent bands were carried out in red hydrogen light. This light is particularly suitable, on account of the bands being separated at a greater distance than with the other available wave-lengths, and also by reason of the particular brilliancy of this radiation afforded by the Riedel-Geissler tube at the ordinary temperature of the tube. The better of the two parallel surfaces of the aluminium block, which was placed uppermost for use, afforded bands which were strictly regular and almost perfectly rectilinear, yielding excellent measurements. The temperature was then raised to the neighbourhood of 70°, recording the transit of bands on the tape, and after a couple of hours' constancy measurements were made at this temperature. After the attainment of another interval, the passage of bands being recorded as before, a final set of measurements were made for a constant temperature in the neighbourhood of 120°.

After cooling overnight, a duplicate series of determinations for three similar temperatures were made next day.

Lastly, after cooling during another night, the exact lengths of the platinum-iridium screws projecting above the height of the points were measured by the aid of the thickness measurer; similarly, the known thickness of the aluminium block was verified.

After these duplicate determinations with the same screw-length, an independent series was taken on another day with a different setting of the screws.

Following are the results obtained :—

Expansion of Aluminium.

SERIES 1.

Thickness of aluminium block, $Lt_1 = 12{\cdot}188$.
Length of screws above table points, $l = 12{\cdot}400$.
Thickness of air-layer, $d = 0{\cdot}212$.
Temperatures, $t_1 = 10°{\cdot}1$, $t_2 = 68°{\cdot}9$, $t_3 = 123°{\cdot}4$.

$$t_2 - t_1 = 58{\cdot}8. \quad t_3 - t_1 = 113{\cdot}3.$$

Pressures, $b_1 = 754$, $b_2 = 754{\cdot}8$, $b_3 = 756$ millims.
Number of transited bands, $f_2 = 30{\cdot}35$, $f_3 = 60{\cdot}32$.
Correction for air refraction, $-0{\cdot}04$, $-0{\cdot}06$.
Corrected number of bands, $f_2' = 30{\cdot}31$, $f_3' = 60{\cdot}26$.
Wave-length of light employed (C hydrogen), $\lambda = 0{\cdot}0006562$. $\frac{1}{2}\lambda = 0{\cdot}0003281$.
Diminution of thickness of air-layer, $f_2' \frac{1}{2}\lambda = 0{\cdot}0099449$. $f_3' \frac{1}{2}\lambda = 0{\cdot}019772$.
Elongation of tripod screws = measured length of screws × ($a + 2bt$ for tripod alloy) × temperature interval,

$$l\left[10^{-9}\left(8600 + 4{\cdot}56\,\frac{t_1 + t_2}{2}\right)\right](t_2 - t_1) = 0{\cdot}0064074.$$

$$l\left[10^{-9}\left(8600 + 4{\cdot}56\,\frac{t_1 + t_3}{2}\right)\right](t_3 - t_1) = 0{\cdot}012510.$$

Expansion of aluminium block = diminution of thickness of air-layer + elongation of screws,

$$Lt_2 - Lt_1 = 0{\cdot}0099449 + 0{\cdot}0064074 = 0{\cdot}0163523.$$

$$Lt_3 - Lt_1 = 0{\cdot}019772 + 0{\cdot}012510 = 0{\cdot}032282.$$

$\theta = 0{\cdot}000\,268\,23$. $\phi = 0{\cdot}000\,000\,125\,31$.
$L_0 = 12{\cdot}1853$.
$a = 0{\cdot}000\,022\,013$. $b = 0{\cdot}000\,000\,010\,284$.

SERIES 2.

Thickness of aluminium block, $Lt_1 = 12{\cdot}188$.
Length of screws above table points, $l = 12{\cdot}400$.
Thickness of air-layer, $d = 0{\cdot}212$.
Temperatures, $t_1 = 8°{\cdot}0$, $t_2 = 70°{\cdot}0$, $t_3 = 120°{\cdot}6$.

$$t_2 - t_1 = 62{\cdot}0. \quad t_3 - t_1 = 112{\cdot}6.$$

Pressures, $b_1 = 755{\cdot}1$, $b_2 = 753{\cdot}7$, $b_3 = 752{\cdot}7$ millims.
Number of transited bands, $f_2 = 32{\cdot}20$, $f_3 = 60{\cdot}22$.

Correction, $-0{\cdot}04$, $-0{\cdot}06$.

Corrected number of bands, $f_2' = 32{\cdot}16$, $f_3' = 60{\cdot}16$.

Wave-length of light employed, $\lambda = 0{\cdot}0006562$. $\frac{1}{2}\lambda = 0{\cdot}0003281$.

Diminution of thickness of air-layer, $f_2' \frac{1}{2}\lambda = 0{\cdot}0105520$. $f_3' \frac{1}{2}\lambda = 0{\cdot}019739$.

Elongation of tripod screws,

$$l\left[10^{-9}\left(8600 + 4{\cdot}56\,\frac{t_1+t_2}{2}\right)\right](t_2 - t_1) = 0{\cdot}0067481.$$

$$l\left[10^{-9}\left(8600 + 4{\cdot}56\,\frac{t_1+t_3}{2}\right)\right](t_3 - t_1) = 0{\cdot}012417.$$

Expansion of aluminium block,

$$Lt_2 - Lt_1 = 0{\cdot}010552 + 0{\cdot}0067481 = 0{\cdot}0173001.$$

$$Lt_3 - Lt_1 = 0{\cdot}019739 + 0{\cdot}012417 = 0{\cdot}032156.$$

$\theta = 0{\cdot}000\,268\,94$. $\phi = 0{\cdot}000\,000\,129\,37$.

$L_0 = 12{\cdot}1858$.

$a = 0{\cdot}000\,022\,070$. $b = 0{\cdot}000\,000\,010\,617$.

Series 3.

Thickness of aluminium block, $Lt_1 = 12{\cdot}188$.

Length of screws above table points, $l = 12{\cdot}812$.

Thickness of air-layer, $d = 0{\cdot}624$.

Temperatures, $t_1 = 9^\circ{\cdot}8$, $t_2 = 67^\circ{\cdot}6$, $t_3 = 118^\circ{\cdot}7$.

$$t_2 - t_1 = 57{\cdot}8. \qquad t_3 - t_1 = 108{\cdot}9.$$

Pressures, $b_1 = 772{\cdot}6$, $b_2 = 771{\cdot}7$, $b_3 = 771{\cdot}9$ millims.

Number of transited bands, $f_2 = 29{\cdot}43$, $f_3 = 57{\cdot}18$.

Correction, $-0{\cdot}11$, $-0{\cdot}18$.

Corrected number of bands, $f_2' = 29{\cdot}32$, $f_3' = 57{\cdot}00$.

Wave-length of light employed, $\lambda = 0{\cdot}0006562$. $\frac{1}{2}\lambda = 0{\cdot}0003281$.

Diminution of thickness of air-layer, $f_2' \frac{1}{2}\lambda = 0{\cdot}0096199$. $f_3' \frac{1}{2}\lambda = 0{\cdot}018702$.

Elongation of tripod screws,

$$l\left[10^{-9}\left(8600 + 4{\cdot}56\,\frac{t_1+t_2}{2}\right)\right](t_2 - t_1) = 0{\cdot}0064992.$$

$$l\left[10^{-9}\left(8600 + 4{\cdot}56\,\frac{t_1+t_3}{2}\right)\right](t_3 - t_1) = 0{\cdot}012407.$$

Expansion of aluminium block,

$$Lt_2 - Lt_1 = 0{\cdot}0096199 + 0{\cdot}0064992 = 0{\cdot}0161191.$$

$$Lt_3 - Lt_1 = 0{\cdot}018702 + 0{\cdot}012407 = 0{\cdot}031109.$$

$\theta = 0{\cdot}000\,268\,58$. $\phi = 0{\cdot}000\,000\,132\,75$.

$L_0 = 12{\cdot}1854$.

$a = 0{\cdot}000\,022\,041$. $b = 0{\cdot}000\,000\,010\,895$.

The three results for aluminium are collated below :—

	a.	b.
Series 1	0·000 022 013	0·000 000 010 284
" 2	22 070	10 617
" 3	22 041	10 895

Mean values, $a = 0{\cdot}000\,022\,041$. $b = 0{\cdot}000\,000\,010\,599$.

The agreement of the results is very gratifying, and leaves no doubt as to the accuracy of the observations. The final result may be expressed as follows :

The mean coefficient of expansion $a + bt$ of the pure aluminium of the compensators, between 0° and t°, is

$$0{\cdot}000\,022\,04 + 0{\cdot}000\,000\,010\,6t, \text{ or } 10^{-8}(2204 + 1{\cdot}06t).$$

The true coefficient of expansion at t°, or the mean coefficient between any two temperatures whose mean is t, $\alpha = a + 2bt$, is

$$\alpha = 0{\cdot}000\,022\,04 + 0{\cdot}000\,000\,021\,2t, \text{ or } = 10^{-8}(2204 + 2{\cdot}12t).$$

The value given by FIZEAU for aluminium for 40° ('Compt. Rend.,' vol. 68, p. 1125 ; and 'Ann. Chim. Phys.,' [4], vol. 8, p. 335), is 0·000 023 13 ; he further gives for the increment, $\Delta\alpha/\Delta t = 2b$, the value 2·29. This increment agrees fairly well with the author's value of 2·12 ; and if the value for 0° is calculated by diminishing 0·000 023 13 by 40 times the increment 2·29, the number 0·000 022 21 is obtained, a figure which also agrees satisfactorily with the author's constant a. The coefficient for 40°, calculated from the author's formula, is 0·000 022 89.

DETERMINATION OF THE EXPANSION OF THE CRYSTAL-COVERING-GLASSES.

For the purpose of determining the expansion of the black glass of the covering-glasses employed with crystals whose surfaces cannot be made to take a polish equal to that of glass, a block of the same glass, 13 millims. thick, was procured. The mode of adjustment and observation was precisely similar to that for the aluminium block. Two series of determinations were made, employing red hydrogen light. In each case the platinum-iridium screws were adjusted so as to leave an air-layer about a quarter of a millimetre thick between the upper black glass surface and the lower surface of the large cover-wedge.

This glass proved to have an expansion coefficient slightly less than that of platinum-iridium, so that the converse was observed of what happens in the case of aluminium ; that is to say, instead of a reduction of the thickness of the air-layer by the approach of the two reflecting surfaces, a slight recession was found to occur. Moreover, as the metallic tripod was affected more rapidly by the rising temperature of the air bath, a recession for several bands was first observed, due to the expansion of the screws; the movement of the bands subsequently became arrested for a considerable interval of time, until, when the badly conducting glass eventually approached the same temperature as the tripod, one or two of the transited bands retraced their steps past the spider-lines, so that the actual number of bands eventually found to have effected their permanent transit was very small, corresponding to close similarity of expansion between the glass and the tripod.

The results obtained were as follows:

Expansion of Black Glass.

Series 1.

Thickness of black glass block, $Lt_1 = 12{\cdot}956$.
Length of screws above table points, $l = 13{\cdot}120$.
Thickness of air-layer, $d = 0{\cdot}164$.
Temperatures, $t_1 = 8^\circ{\cdot}9$, $t_2 = 68^\circ{\cdot}9$, $t_3 = 122^\circ{\cdot}6$.

$$t_2 - t_1 = 60{\cdot}0. \quad t_3 - t_1 = 113{\cdot}7.$$

Pressures, $b_1 = 739{\cdot}2$, $b_2 = 739{\cdot}2$, $b_3 = 739{\cdot}4$.
Number of transited bands, $f_2 = 2{\cdot}83$, $f_3 = 4{\cdot}66$.
Correction for air refraction, $+ 0{\cdot}03$, $+ 0{\cdot}05$.
Corrected number of bands, $f_2' = 2{\cdot}86$, $f_3' = 4{\cdot}71$.
Wave-length of light employed, $\lambda = 0{\cdot}0006562$, $\frac{1}{2}\lambda = 0{\cdot}0003281$.
Increase of thickness of air-layer, $f_2' \frac{1}{2}\lambda = 0{\cdot}00093838$. $f_3' \frac{1}{2}\lambda = 0{\cdot}0015454$.
Elongation of tripod screws,

$$l\left[10^{-9}\left(8600 + 4{\cdot}56\,\frac{t_1 + t_2}{2}\right)\right](t_2 - t_1) = 0{\cdot}0069092.$$

$$l\left[10^{-9}\left(8600 + 4{\cdot}56\,\frac{t_1 + t_3}{2}\right)\right](t_3 - t_1) = 0{\cdot}013275.$$

Expansion of black glass block,

$$Lt_2 - Lt_1 = 0{\cdot}0069092 - 0{\cdot}00093838 = 0{\cdot}00597082.$$

$$Lt_3 - Lt_1 = 0{\cdot}013275 - 0{\cdot}0015454 = 0{\cdot}0117296.$$

$\theta = 0{\cdot}000\,094\,226$. $\phi = 0{\cdot}000\,000\,067\,939$.
$L_0 = 12{\cdot}95514$.
$a = 0{\cdot}000\,007\,273\,3$. $b = 0{\cdot}000\,000\,005\,244\,2$.

SERIES 2.

Thickness of black glass block, $Lt_1 = 12\cdot956$.
Length of screws above table points, $l = 13\cdot221$.
Thickness of air-layer, $d = 0\cdot265$.
Temperatures, $t_1 = 6°\cdot0$, $t_2 = 69°\cdot0$, $t_3 = 119°\cdot1$.

$$t_2 - t_1 = 63. \qquad t_3 - t_1 = 113\cdot1.$$

Pressures, $b_1 = 743$, $b_2 = 745$, $b_3 = 746\cdot5$.
Number of transited bands, $f_2 = 3\cdot24$, $f_3 = 5\cdot19$.
Correction, $+ 0\cdot05$, $+ 0\cdot08$.
Corrected number of bands, $f_2' = 3\cdot29$, $f_3' = 5\cdot27$.
Wave-length of light employed, $\lambda = 0\cdot0006562$, $\frac{1}{2}\lambda = 0\cdot0003281$.
Increase of thickness of air-layer, $f_2' \frac{1}{2}\lambda = 0\cdot0010795$. $f_3' \frac{1}{2}\lambda = 0\cdot0017291$.
Elongation of screws,

$$l\left[10^{-9}\left(8600 + 4\cdot56\frac{t_1 + t_2}{2}\right)\right](t_2 - t_1) = 0\cdot0073052.$$

$$l\left[10^{-9}\left(8600 + 4\cdot56\frac{t_1 + t_3}{2}\right)\right](t_3 - t_1) = 0\cdot0132850.$$

Expansion of black glass block,

$$Lt_2 - Lt_1 = 0\cdot0073052 - 0\cdot0010795 = 0\cdot0062257.$$

$$Lt_3 - Lt_1 = 0\cdot0132850 - 0\cdot0017291 = 0\cdot0115559.$$

$\theta = 0\cdot000\ 093\ 804$. $\phi = 0\cdot000\ 000\ 066\ 894$.
$L_0 = 12\cdot95514$.
$a = 0\cdot000\ 007\ 240\ 7$. $b = 0\cdot000\ 000\ 005\ 163\ 6$.
The two results are compared below, and their mean extracted :—

	a.	*b*.
Series 1	0·000 007 273 3	0·000 000 005 244 2
,, 2	7 240 7	5 163 6

Mean values, $a = 0\cdot000\ 007\ 257\ 0$. $b = 0\cdot000\ 000\ 005\ 203\ 9$.

Hence the mean coefficient of expansion $a + bt$ of the black glass of the crystal-covering-discs, between 0° and t° is

$$0\cdot000\ 007\ 257 + 0\cdot000\ 000\ 005\ 20t,$$
$$\text{or } 10^{-9}(7257 + 5\cdot2t).$$

The true coefficient of expansion at t°, or the mean coefficient between any two temperatures whose mean is t, $\alpha = a + 2bt$, is

$$\alpha = 0\cdot000\ 007\ 257 + 0\cdot000\ 000\ 010\ 4t, \text{ or}$$
$$= 10^{-9}(7257 + 10\cdot4t).$$

The Determination of the Expansion of a Crystal.

It only remains now to briefly indicate the mode of employing the compensator for the determination of the expansion of a parallel-faced block of a crystal or any other substance, whose thickness between the parallel plane surfaces need not exceed 5 millims.

In case the method is followed of employing the compensator below the crystal, one of the two thinnest aluminium compensating blocks, respectively 4 and 6 millims. thick, is employed, together with one of the 2 millims. thick black glass discs of 10 millims. diameter. The exact length of platinum-iridium screw which expands to the same amount as each of these, is readily found from the knowledge now available of the coefficient of expansion of each. Knowing, therefore, the length of screw whose expansion would be exactly counterbalanced by the added expansion of the particular aluminium compensator and crystal-covering-glass, the screws are to be set to this length above the raised points by the aid of the thickness measurer. As regards choice of compensator, that one is selected whose thickness, added to that of the covering-glass, will be less than the corresponding screw-length by an amount which is just slightly greater than the thickness of the crystal block, slightly greater in order to leave room for the necessary air film between the two reflecting surfaces.

The compensator is placed on the raised points of the tripod. A masking disc (lens stop), similar to that referred to in connection with the determinations of the tripod expansion, but of smaller aperture, is next laid on the compensator. Its aperture is slightly less than the diameter of the black covering-glass, so that it arrests all light other than that from the covering-glass. The crystal block is then laid with one of its two plane parallel surfaces resting on the centre of the aluminium block within the aperture of the mask, and the black crystal-covering-glass, polished face upwards, is placed on the top of the crystal. On then laying the large cover-wedge over the screws, its lower surface should be found to be separated by the desired air-layer from the polished surface of the black glass. The whole is then transferred to the interference chamber and the adjustment of the images from the reflecting surfaces proceeded with.

If the method of employing the compensator above the crystal is chosen, the latter is laid first on the three points of the tripod table, and on it is then laid one of the smaller compensators furnished with three points, these latter resting directly on the crystal. No covering-glass is required, the aluminium surface reflecting light adequately. The screw-length should be that calculated for complete compensation by the compensator chosen.

In effecting the adjustment of the lower reflecting surface by slight variation of the screws, it is preferable to do so in such a manner that any difference of screw-length introduced, from that which is compensated, should be an excess. This can readily be done by making the adjustment by screwing further out for the required minute distance one, or, if necessary, two of the screws, rather than by slightly withdrawing the opposite screw. The final measurement of the actual screw-length, after the completion of the observation, affords the information required for the calculation of the exact amount uncompensated. This minute amount is most conveniently taken into account in the calculations in the form of a correction to the change of thickness of the air-layer, or, which is the same thing, amount of expansion or contraction of the crystal. If, as usual, the crystal expands with rise of temperature, the correction is positive; the effect of the slight apparent expansion of the screws being to reduce the observed amount of approach of the two reflecting surfaces; if the crystal is a contracting one along this particular direction, the correction requires to be subtracted, as the effect is to enhance the apparent contraction. The amount of the correction is the difference between the calculated amounts of expansion of the screws and the compensator; these are obtained by multiplying the measured screw-length and the thickness of the aluminium block respectively, by the now-ascertained mean coefficient of expansion of the tripod alloy and aluminium, for each of the two intervals of temperature, and by the number of degrees of rise of temperature, in each case.

The mode of carrying out the determinations of the number of bands which make their transit during the two intervals of temperature is precisely similar to the procedure which has been described for the cases of the determination of the expansion of the aluminium compensator and of the glass of the covering-discs.

In a subsequent memoir, the author hopes to present to the Royal Society the results of a series of determinations, carried out in this manner, of the thermal expansion of the crystals of the sulphates and selenates of potassium, rubidium, and cæsium, an investigation which is now engaging his attention. Further details, therefore, as to the mode of carrying out determinations with crystals, and concerning the preparation of suitable crystal blocks, will be left for consideration in that memoir.

The dilatometer and its accessories have been constructed for the author in an altogether admirable manner by Messrs. Troughton and Simms.

Fig. 6.

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